MATH 210C - FINAL EXAM - TAKE HOME

2009 June 1

Deadline : Friday June 12 at 12:00 pm, noon.

Question 1. Show that a commutative ring A is artinian if and only if it is noetherian and $\mathrm{Spec}(A) = \mathrm{Max}(A)$ (i.e. it has Krull dimension zero). Does left artinian imply left noetherian for non-commutative rings too ?

Question 2. Show that a local noetherian domain A of dimension one (i.e. $\mathrm{Spec}(A) = \{0, \mathfrak{m}\}$ with $\mathfrak{m} \neq 0$) which is integrally closed is a DVR.

Question 3. Let G be a finite group of exponent m. Let K be a field whose characteristic does not divide the order of G (for instance $\mathrm{char}(K) = 0$). Is it enough for K to contain a primitive m-th root of unity to be a splitting field of G ?

Question 4. Let G be a finite group whose character table contains the following two rows :

$$\begin{array}{llllllll} \chi_1 : & 1 & 1 & 1 & \omega^2 & \omega & \omega^2 & \omega \\ \chi_2 : & 2 & -2 & 0 & -1 & -1 & 1 & 1 \end{array}$$

where ω is a primite cubic root of unity. Determine the rest of the character table. Give as much information on G as you can.

Question 5. Since S_3 and S_4 are solvable, what is the general solution by radicals of a polynomial equation of degree three and four ? (Give formulas.)