Solutions to Problems : Chapter 19
Problems appeared on the end of chapter 19 of the Textbook

8. **Picture the Problem:** Two point charges exert an electrostatic force on each other.

   Strategy: Solve Coulomb's law (equation 19-5) for the separation distance r.

   Solution: Solve equation 19-5 for r $r=\sqrt{\dfrac{kq_1q_2}{F}}=\sqrt{\dfrac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(11.2\times10^{-6}\,\text{C}\right)\left(29.1\times10^{-6}\,\text{C}\right)}{1.77\,\text{N}}}=\boxed{1.29\text{ m}}$

   Insight: Although 1.77 N is only about 6.4 ounces of force, these microcoulomb-sized charges can exert such a force over the substantial distance of 1.29 m or 4.23 ft.

28. **Picture the Problem:** The honeybee acquires an electrostatic charge in active flight.

    Strategy: Use the magnitude of an electron's charge e to find the number of electrons that correspond to the 93.0 pC total charge. Then use Coulomb's law to find the magnitude of the force between the two charged bees, and compare it with the weight of a single bee.

    Solution: 1. (a) Find the number of electrons: $N=\dfrac{Q}{e}=\dfrac{93.0\times10^{-12}\text{ C}}{1.60\times10^{-19}\text{ C/electron}}=\boxed{5.81\times10^{8}\text{ electrons}}$

    2. (b) Use Coulomb's law to find the force between two charged bees: $F=\dfrac{kq_1q_2}{r^2}=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{\left(93.0\times10^{-12}\text{ C}\right)^2}{\left(0.0120\text{ m}\right)^2}$

    $=\boxed{5.40\times10^{-7}\text{ N}}$

    3. Determine the weight of a bee: $W=mg=\left(0.140\times10^{-3}\text{ kg}\right)\left(9.81\text{ m/s}^2\right)=\underline{\underline{1.37\times10^{-3}\text{ N}}}$

    4. Calculate the ratio of the forces: $\dfrac{F}{mg}=\dfrac{5.40\times10^{-7}\text{ N}}{1.37\times10^{-3}\text{ N}}=3.94\times10^{-4}=\boxed{\dfrac{1}{2540}}$

    Insight: The electrical force between the bees is a tiny fraction of their weight because the amount of electrical charge is quite small. It would require a charge of only 4.68 nC on each bee for the electrical force to equal the weight!

32. **Picture the Problem:** An electric field exists around a 5.00 μC charge at the origin.

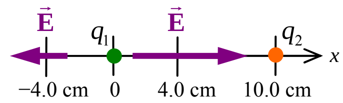
    Strategy: Use the definition of the electric field (equation 19-10) to determine its magnitude.

    Solution: 1. (a) Apply equation 19-10 directly: $E=\dfrac{kq}{r^2}=\dfrac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\,\text{C}\right)}{\left(1.00\,\text{m}\right)^2}=\boxed{4.50\times10^{4}\text{ N/C}}$

    2. (b) Repeat for the new distance: $E=\dfrac{kq}{r^2}=\dfrac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\,\text{C}\right)}{\left(2.00\,\text{m}\right)^2}=\boxed{1.12\times10^{4}\text{ N/C}}$

    Insight: When the distance from the charge is doubled, the field is cut to a fourth.

34. **Picture the Problem:** Two charges are placed on the x-axis as shown at right and create an electric field in the space around them.



Strategy: Use equation 19-8 to find the magnitude and direction of the electric fields created by each of the two charges at the specified locations, then find the vector sum of those fields to find the net electric field. At $x = -4.0$ cm the field from q_1 will point in the $-\hat{\mathbf{x}}$ direction and the field from q_2 will point in the $+\hat{\mathbf{x}}$ direction.

Solution: 1. (a) Sum the fields produced by the two charges at $x = -4.0$ cm:

$$\vec{\mathbf{E}} = \frac{k|q_1|}{r_1^2}(-\hat{\mathbf{x}}) + \frac{k|q_2|}{r_2^2}\hat{\mathbf{x}} = k\left(-\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2}\right)\hat{\mathbf{x}}$$

$$= \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[-\frac{6.2\times10^{-6}\,\text{C}}{(0.040\,\text{m})^2} + \frac{9.5\times10^{-6}\,\text{C}}{(0.140\,\text{m})^2}\right]\hat{\mathbf{x}} = \boxed{\left(-3.0\times10^7\ \text{N/C}\right)\hat{\mathbf{x}}}$$

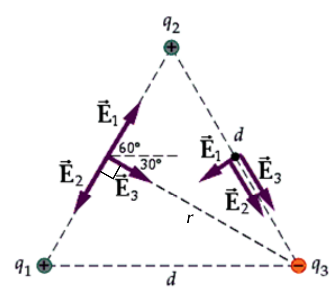
2. **(b)** Repeat for $x = 4.0$ cm:

$$\vec{\mathbf{E}} = \frac{k|q_1|}{r_1^2}\hat{\mathbf{x}} + \frac{k|q_2|}{r_2^2}\hat{\mathbf{x}} = k\left(\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2}\right)\hat{\mathbf{x}}$$

$$= \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{6.2\times10^{-6}\,\text{C}}{(0.040\,\text{m})^2} + \frac{9.5\times10^{-6}\,\text{C}}{(0.060\,\text{m})^2}\right]\hat{\mathbf{x}} = \boxed{\left(5.9\times10^7\ \text{N/C}\right)\hat{\mathbf{x}}}$$

Insight: Although q_2 has a larger magnitude than q_1, at $x = -4.0$ cm the closer distance to q_1 means its contribution to the field is larger than the contribution from q_2, and the net field points in the $-\hat{\mathbf{x}}$ direction.

36. **Picture the Problem:** Three charges are positioned as shown at right.



Strategy: Each of the three charges produces its own electric field that surrounds it. The total electric field at any point is the vector sum of the fields from each charge. Use equation 19-10 and the component method of vector addition to find the magnitude electric field at the points indicated in the problem statement. Let q_1 be at the origin and q_3 be on the positive x-axis.

Solution: 1. (a) At a point halfway between charges q_1 and q_2 the vectors $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ cancel one another. The remaining contribution comes from q_3. First find the distance r from q_3 to the midpoint of the opposite side:

$$r^2 + (d/2)^2 = d^2$$

$$r = \sqrt{3d^2/4}$$

$$= \sqrt{3(0.0275\text{ m})^2/4}$$

$$r = \underline{\underline{0.0238\text{ m}}}$$

2. Apply equation 19-10 to find E_3:

$$E_3 = \frac{kq_3}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\ \text{C}\right)}{(0.0238\text{ m})^2} = \boxed{7.94\times10^7\ \text{N/C}}$$

3. (b) At this location, the electric fields of q_2 and q_3 add, and the resulting field points toward q_3. The field due to q_1 will have the same magnitude as found in part (a) and will be perpendicular to the combined fields of q_2 and q_3. The vector sum of the electric fields from all three charges will have a magnitude $\boxed{\text{greater than}}$ that found in part (a).

4. (c) Find the components of $\vec{\mathbf{E}}_1$:

$$\vec{\mathbf{E}}_1 = \frac{k|q_1|}{3d^2/4}\left(\cos 30°\,\hat{\mathbf{x}} + \sin 30°\,\hat{\mathbf{y}}\right) = \frac{k|q_1|}{d^2}\left(\frac{2\sqrt{3}}{3}\hat{\mathbf{x}} + \frac{2}{3}\hat{\mathbf{y}}\right)$$

5. Find the components of $\vec{\mathbf{E}}_2$:

$$\vec{\mathbf{E}}_2 = \frac{k|q_2|}{(d/2)^2}\left(\cos 60°\,\hat{\mathbf{x}} - \sin 60°\,\hat{\mathbf{y}}\right) = \frac{k|q_2|}{d^2}\left(2\,\hat{\mathbf{x}} - 2\sqrt{3}\,\hat{\mathbf{y}}\right)$$

6. Find the components of $\vec{\mathbf{E}}_3$:

$$\vec{\mathbf{E}}_3 = \frac{k|q_3|}{(d/2)^2}\left(\cos 60°\,\hat{\mathbf{x}} - \sin 60°\,\hat{\mathbf{y}}\right) = \frac{k|q_3|}{d^2}\left(2\,\hat{\mathbf{x}} - 2\sqrt{3}\,\hat{\mathbf{y}}\right)$$

7. Let $|q_1| = |q_2| = |q_3| = q$ and find the vector sum:

$$\vec{\mathbf{E}}_{\text{net}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3$$

$$\vec{\mathbf{E}}_{\text{net}} = \frac{kq}{d^2}\left[\left(4 + \frac{2\sqrt{3}}{3}\right)\hat{\mathbf{x}} + \left(\frac{2}{3} - 4\sqrt{3}\right)\hat{\mathbf{y}}\right]$$

8. Determine the magnitude of E_{net} :

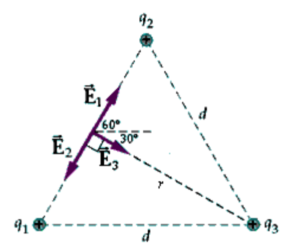
$$E_{\text{net}} = \frac{kq}{d^2}\sqrt{\left(4 + \frac{2\sqrt{3}}{3}\right)^2 + \left(\frac{2}{3} - 4\sqrt{3}\right)^2}$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\ \text{C}\right)}{\left(0.0275\ \text{m}\right)^2}(8.110)$$

$$E_{\text{net}} = \boxed{4.82\times10^8\ \text{N/C}}$$

Insight: As expected, the field is larger at the point midway between q_2 and q_3, about six times larger in magnitude than at the point midway between q_1 and q_2.

38. **Picture the Problem:** Three identical charges are placed as shown in the figure at the right.



Strategy: Each of the three charges produces its own electric field that surrounds it. The total electric field at any point is the vector sum of the fields from each charge. As illustrated in the figure, at the midpoint of any of the three sides of the triangle two of the three $\vec{\mathbf{E}}$ vectors will cancel. Use equation 19-10 to find the magnitude electric field at the midpoints by finding the magnitude of the third, unbalanced vector.

Solution: 1. (a) First find the distance *r* from q_3 to the midpoint of the opposite side:

$$r^2 + (d/2)^2 = d^2$$

$$r = \sqrt{3d^2/4} = \sqrt{3(0.21\ \text{m})^2/4} = \underline{\underline{0.18\ \text{m}}}$$

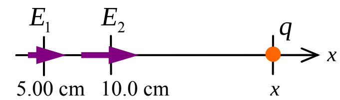
2. Now apply equation 19-10:

$$E = \frac{kq}{r^2} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(4.7\times10^{-6}\,\text{C}\right)}{(0.18)^2} = \boxed{1.3\times10^6\ \text{N/C}}$$

3. (b) Due to symmetry, the three electric field vectors at the center of the triangle cancel out and the net field there is zero. So, the magnitude there is $\boxed{\text{less than}}$ that at the midpoint of a side.

Insight: Whenever identical charges are arranged in a symmetrical fashion, the electric field at their geometric center is zero.

40. **Picture the Problem:** The field produced by an unknown charge at an unknown position is given at two points, as depicted in the figure at the right.



Strategy: Because both measurements of the electric field point in the positive *x*-direction, the point charge must be on the *x*-axis. Also, since the magnitude of an electric field is larger the closer it is measured to its source, the position at the point charge must be greater than $x = 10.0$ cm. Furthermore, since the vectors point toward the right, they must point toward the charge and the charge must be negative. Write out equation 19-10 for the two given fields and combine them to get a quadratic equation in *x*. Solve the expression to find the location of the charge. Then use either of the two field equations to find the magnitude of the charge.

Solution: 1. (a) Use equation 19-10 to express the field magnitudes:

$$E_1 = \frac{k|q|}{(x-x_1)^2} \quad \text{and} \quad E_2 = \frac{k|q|}{(x-x_2)^2}$$

2. Combine the equations to eliminate *q*, take the square root of both sides, and solve for *x*:

$$E_1(x-x_1)^2 = kq = E_2(x-x_2)^2$$

$$\sqrt{E_1}(x-x_1) = \sqrt{E_2}(x-x_2)$$

$$x = \frac{\sqrt{E_1}x_1 - \sqrt{E_2}x_2}{\sqrt{E_1}-\sqrt{E_2}}$$

$$= \frac{\sqrt{10.0\text{ N/C}}(5.00\text{ cm}) - \sqrt{15.0\text{ N/C}}(10.0\text{ cm})}{\sqrt{10.0\text{ N/C}} - \sqrt{15.0\text{ N/C}}} = \boxed{32\text{ cm}}$$

3. (b) Find the magnitude of *q* from E_1:

$$|q| = \frac{E_1(x-x_1)^2}{k} = \frac{(10.0\text{ N/C})(0.32-0.0500\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 8.1\times10^{-11}\text{ C} = 81\text{ pC}$$

4. Because the field vector points toward the charge it must be negative: $q = \boxed{-81\text{ pC}}$

Insight: If the charge were positive, the field magnitudes would be the same but the vectors would point toward −*x*. The rules of subtraction limit the answers to only two significant figures.

56. **Picture the Problem:** A proton is accelerated from rest in a uniform electric field and travels along a straight line.

Strategy: Use Newton's Second Law to find the acceleration of the proton from the electric force $F = qE$ exerted by the electric field. Once the acceleration is known we can find the speed of the particle from the equation for velocity as a function of position, $v^2 = v_0^2 + 2a\Delta x$ (equation 2-12).

Solution: 1. (a) Apply Newton's Second Law to find *a*:

$$\sum F = qE = ma \quad \Rightarrow \quad a = qE/m$$

2. Solve equation 2-12 for *v* when $v_0 = 0$:

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{0 + 2(qE/m)\Delta x}$$

3. Find *v* for $\Delta x = 0.0100$ m:

$$v = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(1.08\times10^5\text{ N/C})(0.0100\text{ m})}{1.673\times10^{-27}\text{ kg}}} = \boxed{4.55\times10^5\text{ m/s}}$$

4. (b) Find *v* for $\Delta x = 0.100$ m:

$$v = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(1.08\times10^5\text{ N/C})(0.100\text{ m})}{1.673\times10^{-27}\text{ kg}}} = \boxed{1.44\times10^6\text{ m/s}}$$

Insight: The large charge-to-mass ratio of elementary particles produces large accelerations when they are immersed in an electric field. In this case the acceleration of the proton is an astounding 1.03×10^{13} m/s^2!

58. **Picture the Problem:** A point charge situated at the origin produces an electric field that completely surrounds it.

Strategy: Use the definition of the electric field (equation 19-10) to find the magnitude of the charge that creates the stipulated electric field. Since the electric field points toward the origin and therefore toward the charge, the sign of the charge must be negative.

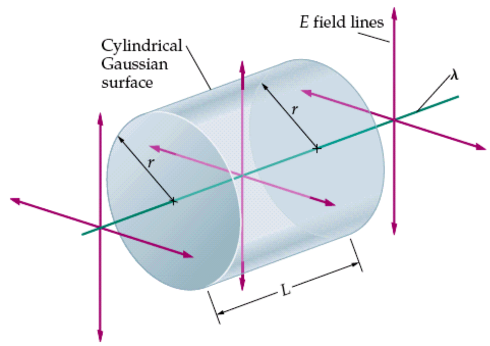
Solution: Solve equation 19-10 for q, assuming the charge is negative:

$$|q| = \frac{Er^2}{k} = \frac{(36{,}000 \text{ N/C})(0.50 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 1.0\times10^{-6} \text{ C}$$

$$q = \boxed{-1.0\ \mu\text{C}}$$

Insight: If the sign of the charge had been positive, the field would have pointed in the $-\hat{\mathbf{x}}$ direction.

64. **Picture the Problem:** A cylindrical Gaussian surface is built around a long, thin wire that has a charge per unit length λ on it.



Strategy: In problem 55 we used Gauss's law to find that

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where r is the distance from the wire and λ is the wire's charge per unit length. Use this expression and the given information to find λ, and then use a ratio to find r when E is reduced by a factor of 2.

Solution: 1. (a) Solve $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ for λ:

$$\lambda = 2\pi\varepsilon_0 rE$$

$$= 2\pi\left(8.85\times10^{-12} \text{ C}^2/\text{ N}\cdot\text{m}^2\right)(0.477 \text{ m})(25{,}400 \text{ N/C}) = \boxed{6.74\times10^{-7} \text{ C/m}}$$

2. **(b)** Use a ratio to find the new r:

$$\frac{r_{\text{new}}}{r_{\text{old}}} = \frac{\lambda/2\pi\varepsilon_0 E_{\text{new}}}{\lambda/2\pi\varepsilon_0 E_{\text{old}}} = \frac{E_{\text{old}}}{E_{\text{new}}} = \frac{E_{\text{old}}}{\frac{1}{2}E_{\text{old}}} = 2$$

$$r_{\text{new}} = 2r_{\text{old}} = 2(0.477 \text{ m}) = \boxed{0.954 \text{ m}}$$

Insight: Another way to solve part (b) is to use the λ from part (a) to solve the given expression for r. However, using a ratio can often be a time-saving step that also avoids the accumulation of rounding errors that can occur.