Chapter 24 Alternating-Current Circuit

Outline

24-1 Alternating Voltage and Circuit

In an alternating circuit, the magnitude and direction of the voltage and current change periodically, and they are a function of time:

The current and voltage is the so-called alternating current (AC) and voltage, respectively.

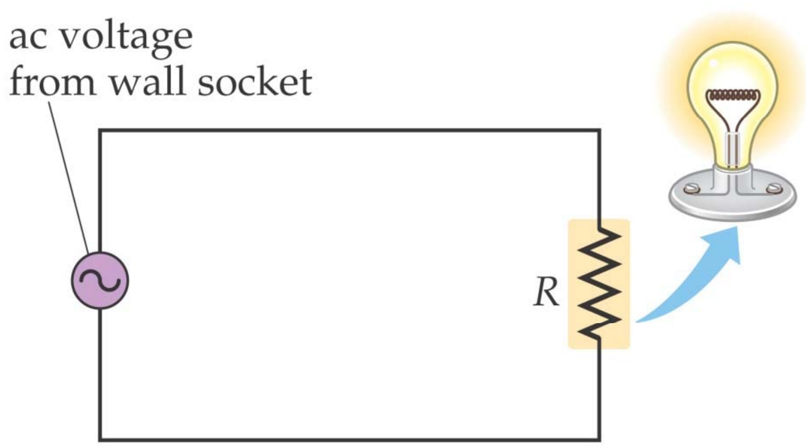


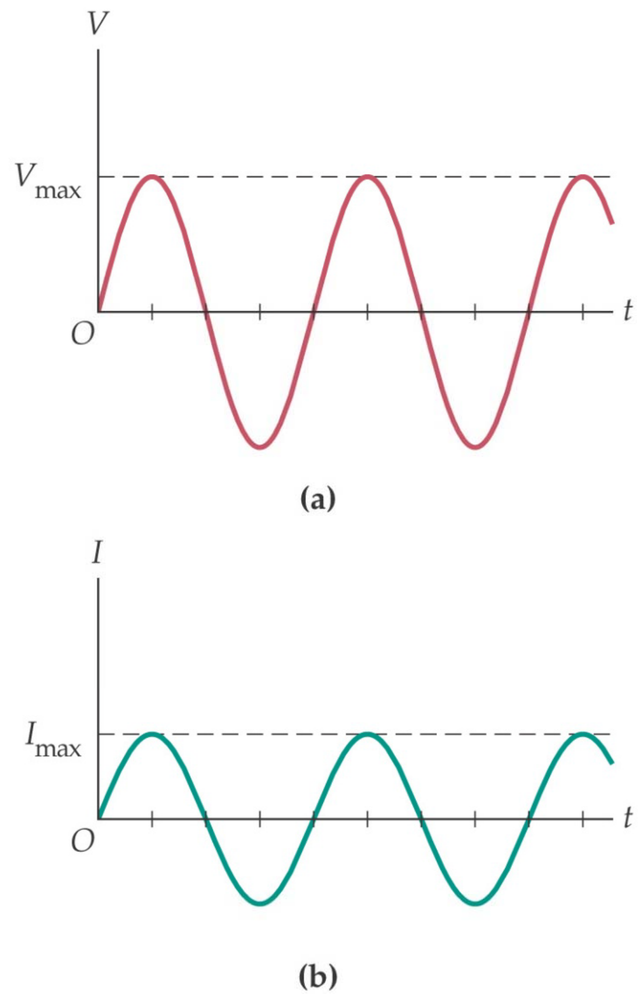
Figure 24-1
An AC Generator Connected to a Lamp

$$V = V_{max} \sin \omega t \qquad (24-1)$$

Where, $\omega = 2\pi f$, (f = 60 Hz).

According to Ohm's law,

$$I = \frac{V}{R} = \frac{V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$



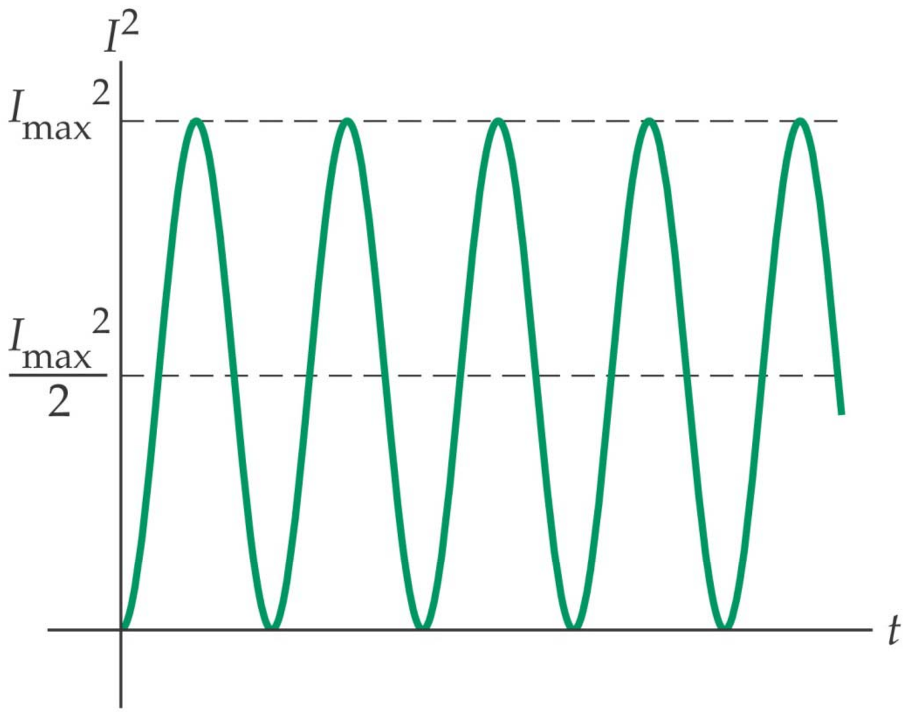
Root Mean Square (rms) Value

Both alternating voltage and current have a zero value. So direct average gives no information (or useless).

In order to evaluate an alternating parameter in quantity, we use root mean square (rms):

We square the alternating current I,

$$I^2 = I^2_{\max} \sin^2 \omega t$$



Now, we can average I^2,

$$(I^2)_{av} = \frac{1}{2} I^2_{\max}$$

RMS is square root of the above eq.,

$$I_{rms} = \frac{1}{\sqrt{2}} I_{\max}$$

Rms: Square, average, square root

Any quantity x that varies with time as $x = x_{max} \sin \omega t$, or *$x = x_{max} \cos \omega t$, obey the relationships:*

RMS Value of a Quantity with Sinusoid Time Dependence

$$(x^2)_{av} = \frac{1}{2} x_{\max}^2$$

$$x_{rms} = \frac{1}{\sqrt{2}} x_{\max} \qquad (24-4)$$

So, the rms value of the voltage in a AC circuit is

$$V_{rms} = \frac{1}{\sqrt{2}} V_{\max} \qquad (24-5)$$

Also suitable for current !

Exercise 24-1

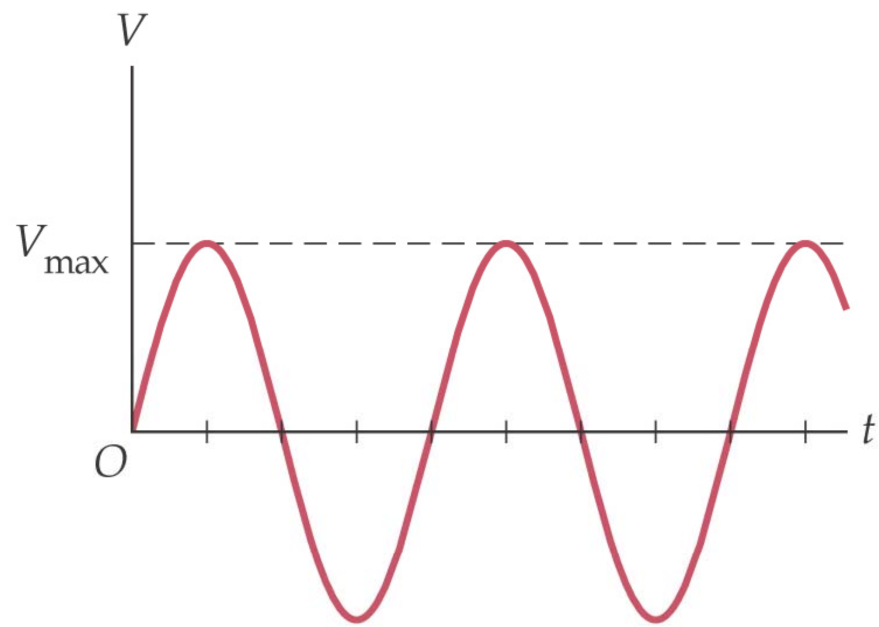
Typical household circuit operates with an rms voltage of 120V. What is the maximum, or peak value of the voltage in the circuit?

Solution

Since $V_{rms} = \frac{1}{\sqrt{2}} V_{\max}$, we have

The maximum and peak is:

$$V_{\max} = \sqrt{2} V_{rms} = \sqrt{2}(120V) = 170 \quad V$$



"Average" Power

Since $P = I^2 R$,

Replace I with I_{rms}, we have the average value of P

$$P_{av} = I_{rms}^2 R$$

Apply Ohm's law,

$$P_{av} = \frac{V_{rms}^2}{R} \qquad (24-6)$$

Rms can operate directly for Ohm law !

A Resistor Circuit

An AC generator with a maximum voltage of 24.0 V and a frequency of 60.0 Hz is connected to a resistor with a resistance R= 265 Ω.

Find (a) the rms voltage and (b) the rms current in the circuit. Determine (c) the average and (d) maximum power dissipated in the resistor.

Solution

(a) The rms voltage is

$$V_{rms} = \frac{1}{\sqrt{2}} V_{\max} = \frac{1}{\sqrt{2}}(24.0) = 17.0 \quad V$$

(b) The rms current is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{17.0V}{265\Omega} = 0.0642 \quad A$$

(c) The average power

$$P_{av} = \frac{V^2{}_{rms}}{R} = \frac{(17.0V)^2}{265\Omega} = 1.09 \quad W$$

(d) The maximum power

$$P_{\max} = \frac{V^2{}_{\max}}{R} = 2\frac{V^2{}_{rms}}{R} = 2 \times 1.09 \quad W = 2.18 \quad W$$

CONCEPTUAL CHECKPOINT 24–1

If the frequency of the ac generator in Example 24–1 is increased, does the average power dissipated in the resistor **(a)** increase, **(b)** decrease, or **(c)** stay the same?

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Reasoning and Discussion

None of the results in Example 24–1 depend on the frequency of the generator. For example, the relation $V_{rms} = V_{max}/\sqrt{2}$ depends only on the fact that the voltage varies sinusoidally with time, and not at all on the frequency of the oscillations. The same frequency independence applies to the rms current and the average power.

These results are due to the fact that resistance is independent of frequency. In contrast, we shall see later in this chapter that the behavior of capacitors and inductors does indeed depend on frequency.

Answer:

(c) The average power remains the same.

Homework Problems: 2, 4

2. In many European homes the rms voltage available from a wall socket is 240 V. What is the maximum voltage in this case?

4. The rms current in an ac circuit with a resistance of 150 Ω is 0.85 A. What are the (a) average and (b) maximum power consumed by this circuit?

Summary

The calculate of RMS for a $x=x_{max}sin\omega t$,

or $x=x_{max}cos\omega t$, function