March 25, 2010 **MATH 351, Spring 2010** Prof. V. Panferov

Homework Assignment 6

Due Thu. April. 1, 2010, in class.

1. Problems 4, 7, 8, Section 3.5, pp. 122–124.

2. Transform the following higher-order equations into a system of first order, and write the system in matrix form $x' = Ax$

   (a) $u'' + \frac{1}{2}u' + 2u = 3\sin t$

   (b) $t^2u'' + tu' + (t^2 - \frac{1}{4})u = 0$

3. Write the system of equations for the two mass-spring system with two masses,

$$\begin{cases} m_1 x_1'' = -(k_1 + k_2)x_1 + k_2 x_2 \\ m_2 x_2'' = k_2 x_1 - (k_2 + k_3)x_2 \end{cases}$$

   in the form of a first order system $x' = Ax$.

4. Systems of first order equations can sometimes be transformed into a single equation of higher order. Consider the system

$$\begin{cases} x_1' = -2x_1 + x_2 \\ x_2' = x_1 - 2x_2. \end{cases}$$

   (a) Solve the first equation for x_2 and substitute into the second equation, thereby obtaining a second order equation for x_1. Solve this equation for x_1 and then determine x_2 also.

   (b) Find the solution for the given system that also satisfies the intial conditions $x_1(0) = 2$, $x_2(0) = 3$.

   (c) Sketch the curve, for $t \geq 0$, given parametrically by the expressions for $x_1(t)$ and $x_2(t)$ obtained in part (b).

5. Same as problem 4 for the system

$$\begin{cases} x_1' = 2x_2, & x_1(0) = 3 \\ x_2' = -2x_1, & x_2(0) = 4. \end{cases}$$

6. Problems 4 (a) (d) (h), 5, 6, 12, Section 5.3, pp. 196–198.