March 23, 2010 **MATH 351, Spring 2010** Prof. V. Panferov

Homework Assignment 4

Due Thu. Mar. 18, 2010, in class.

1. Problems 3, 8, 9, 10, 12, section 3.2 (pp. 94-95).

2. Show that a particular solution of the equation

$$u'' + pu' + qu = f_1(t) + f_2(t)$$

can be found in the form $u_p(t) = u_1(t) + u_2(t)$ where $u_1(t)$ and $u_2(t)$ satisfy

$$u_1'' + pu_1' + qu_1 = f_1(t) \quad \text{and} \quad u_2'' + pu_2' + qu_2 = f_2(t).$$

3. Find the general solutions to the following nonhomogeneous equations:

   (a) $u'' - u' = 6 + e^{2t}$

   (b) $u'' - 3u' - 4u = 2t^2 + te^t + 3\sin t$

   (c) $u'' - 4u = \cos(2t)$

   (d) $u'' + 4u = \cos(2t)$

   (e) $u'' + u' + 2u = \sin^2 t$

4. Use the method of variation of parameters to find general solutions of nonhomogeneous equations:

   (a) $u'' + u = \tan t$, $0 < t < \frac{\pi}{2}$

   (b) $u'' - 2u' + u = e^t/(1 + t^2)$

5. Solve the intial-value problem $u'' - 3u' - 40u = 2e^{-t}$, $u(0) = 0$, $u'(0) = 1$.

6. Compute the Wronski determinant $\Delta(t)$ for the functions $u_1(t) = e^t$, $u_2(t) = \sin t$ and show that $\Delta(\frac{\pi}{4}) = 0$. Explain why there is no contradiction with the properties of the Wronskian discussed in class ($\Delta(t)$ is either zero for all t, or never zero).

7. A circuit contains a 10^{-3} farad capacitor in series with a 20 volt battery and an inductor of 0.4 henrys. At $t = 0$ both $q = 0$ and $I = 0$. Find the charge $q(t)$ on the capacitor and describe the response of the circuit in terms of transients and steady states.