February 23, 2010 **MATH 351, Spring 2010** Prof. V. Panferov

Homework Assignment 3

Due Tue. Feb. 23, 2010, in class.

In the following two problems y denotes the unknown function, and x the independent variable.

1. Solve the following linear equations

   (a) $xy' - 2y = 2x^4$

   (b) $y' + y \tan x = \sec x$

   (c) $(xy' - 1) \ln x = 2y$

2. Solve the Bernoulli equations (see Problem 10 in 2.2 discussed in class)

   (a) $u' = u(1 + ue^t)$

   (b) $u' = -\frac{1}{t}u + \frac{1}{tu^2}$

   (c) $y' + 2y = y^2 e^x$

3. (Problem 9, 2.1) A differential equation of the form

   $$u' = F(\tfrac{u}{t})$$

   where the right depends only on the ratio of u and t, is called *homogeneous*. Show that the substitution $u = ty$ transforms a homogeneous equation into a first-order separable equation for $y = y(t)$. Use this method to solve the equations

   (a) $u' = \frac{4t^2+3u^2}{2tu}$

   (b) $tu' = u - te^{u/t}$

   (c) $tu' - u = t \tan \frac{u}{t}$

4. (Problem 12, 2.2) Initially, a tank contains 60 gal of pure water. Then brine containing 1 lb of salt per gallon enters the tank at 2 gal/min. The perfectly mixed solution is drained off at 3 gal/min. Determine the amount (in lbs) of salt in the tank up until the time it empties.

5. (Problem 18, 2.2) In a community having a fixed population N, the rate that people hear a rumor is proportional to the number of people who have not yet heard the rumor. Write down a DE for the number of people P who have heard the rumor. Over a long time, how many will hear the rumor?

6. (Problem 1, 2.3.1) Consider the initial value problem

$$u' = 1 + u^2, \quad u(0) = 0.$$

Apply Picard iteration with $u_0 = 0$ and compute four terms. If the process continues, to what function will the resulting series converge?

7. (Problem 2, 2.3.1) Apply Picard iteration to the initial value problem

$$u' = t - u, \quad u(0) = 1,$$

to obtain three Picard iterates, taking $u_0 = 1$. Plot each iterate and the exact solution on the same set of axes.

8. (Problem 4, 2.3.2) Consider the initial value problem for the decay equation,

$$u' = -ru, \quad u(0) = u_0.$$

Here, r is a given positive decay constant. Find the exact solution to the initial value problem and the exact solution to the sequence of difference approximations $u_{n+1} = u_n - hru_n$ defined by the Euler method. Does the discrete solution give a good approximation to the exact solution for all stepsizes h? What are the constraints on h?

9. (Problem 7, 2.3.2) Consider the initial value problem

$$u' = 5u - 6e^{-t}, \quad u(0) = 1.$$

Find the exact solution and plot it on the interval $0 \le t \le 3$. Next use the Euler method with $h = 0.1$ to obtain a numerical solution. Explain the results of this numerical experiment.

10. (Variation of Problem 7, 2.1) Find the general solution of the DE

$$3(1 - t^2)u' + 2tu = 2t.$$

Find the particular solutions that satisfy the initial conditions

(a) $u(0) = 0$

(b) $u(3) = -1$

(c) $u(0) = 1$.

What are the maximal intervals of existence for each of these solutions? Sketch the graphs of these solutions on the same set of axes.