1. Solve $y'' - 4y = 5e^{2x}$.

2. Solve the system
$$y_1' = -2y_1 - 2y_2$$
$$y_2' = -2y_1 + y_2$$

3. Consider the initial value problem $y' = xy^2 + 1,\ y(0) = 0$ on the rectangle
$$R = \{(x, y):\ -3 \le x \le 3,\ -4 \le y \le 4\}.$$
   (a) Give the equivalent integral equation.
   (b) Compute the successive approximations ϕ_0, ϕ_1, ϕ_2.
   (c) Show $f(x, y) = xy^2 + 1$ satisfies a Lipschitz condition on R.
   (d) Using R, find an interval I on which the sequence $\phi_0, \phi_1, \phi_2, \ldots$ must converge to a solution ϕ of the initial value problem.

4. Use Euler's method with $h = 1$ to find an approximate solution on the interval $[0, 3]$ of the initial value problem
$$y' = xy^2 + 1,\ y(0) = 0.$$
Also, draw the graph of your approximate solution.

5. Solve the initial value problem $y' = \sqrt{x} - \frac{y}{x},\ y(1) = 4$.

6. Find the value of the constant k that makes $(x^2 \sin y - 1)y' = -3x^2 + kx \cos y$ an exact equation, and then solve the equation.

7. Solve $x^2y'' + 4xy' - 10y = 7x^{-5},\ x > 0$.

8. Find one non-trivial solution of $x^2y'' + x^2y' - 6y = 0,\ x > 0$. (If your solution involves a series, you can write the series in the form: first 3 non-zero terms $+ \cdots$.)

9. For each of the following, prove or give a counterexample :
   (a) If the functions f, g are linearly dependent on $[0, 1]$ and are linearly dependent on $[1, 2]$, then f, g are linearly dependent on $[0, 2]$.
   (b) If ϕ_1, ϕ_2 are solutions of $y'' + a_1(x)y' + a_2(x)y = 0$ on $[0, 2]$, and if ϕ_1, ϕ_2 are linearly dependent on $[0, 1]$ and are linearly dependent on $[1, 2]$, then ϕ_1, ϕ_2 are linearly dependent on $[0, 2]$.