Lecture #5   **chapter 5**   **Discrete Probability Distributions**

5-2 Random Variables

Def: A **random variable**, x, represents a numerical value, determined by chance, assigned to an outcome of a probability experiment.

A **Probability distribution** is a graph, table, or formula that gives the probability for each value of the random variable.

Example 1: A sociologist surveyed the households in a small town. The random variable x represents the number of dependent children in the households.

X	0	1	2	3	4
P(x)	0.07	0.20	0.38	0.22	0.13

Types of random variables:

A **discrete** random variable has either a finite number of values or a countable number of values.

A **continuous** random variable has infinitely many values, and those values can be associated with measurements on a continuous scale in such a way that there are no gaps or interruptions.

Example 2: Decide whether the random variable, x, is discrete or continuous.

a) x represents the length of time it takes to complete a test.
b) x represents the number of home runs hit during a baseball game.
c) The speed of the next car that passes a State trooper.
d) The number of A's earned in this class with 40 students enrolled.

Requirements for a Probability Distribution

1. $\sum P(x) = 1$ where x assumes all possible values
2. $0 \leq P(x) \leq 1$ for every value of x.

Example 3: Does the example 1 describe a probability distribution?

Example 4: Decide whether the following distribution is a probability distribution.

X	5	6	7	8
P(x)	1/2	¼	1/2	-1/4

In working with prob. dist., it is useful to have a concept of the typical or average value that the random variable takes on. We now, introduce methods for identifying the center and spread of a random variable.

Mean for a probability distribution: $\mu = \Sigma[x \cdot P(x)]$ (Expected value)

Variance for a probability distribution: $\sigma^2 = \Sigma(x - \mu)^2 . P(x)$

Standard deviation for a probability distribution: $\sigma = \sqrt{\Sigma(x - \mu)^2 . P(x)}$ or
$\sigma = \sqrt{\Sigma[x^2 . P(x)] - \mu^2}$

Round-off rules for μ, σ, σ^2: Round results by carrying one more decimal place than the number of decimal places used for the random variable x. If the values of x are integers, round to one decimal place.

Example 5: To settle a paternity suit, two different people are given blood tests. If x is the number having group A blood, then x can be 0, 1, or 2, and the corresponding probabilities are 0.36, 0.48, and 0.16 respectively. Find its mean and standard deviation.

Example 6: Three males with an X-linked genetic disorder have one child each. The random variable x is the number of children among the three who inherit the X-linked genetic disorder. Find its mean and standard deviation.

X	0	1	2	3
P(x)	0.4219	0.4219	0.1406	0.0156

Identifying unusual results with Range Rule of Thumb: According to the range rule of thumb we can identify "unusual" values be determining that they lie outside of these limits.

Max. usual value= $\mu + 2\sigma$

Min. usual value= $\mu - 2\sigma$

Example 7: find the min. and max. usual values in example 6.

Example 8: Pg 212, #21

Identifying unusual results with Rare Event Rule:

Rare Event Rule: If, under a given assumption (such as the assumption that a coin is fair), the probability of particular observed event (such as 992 heads in 1000 tosses of a coin) is extremely small, we conclude that the assumption is probably not correct.

Unusually High number of successes: x successes among n trial is an unusually high number of successes if P(x or more)≤ 0.05

Unusually low number of successes: x successes among n trial is an unusually low number of successes if P(x or fewer)≤ 0.05.

Example 9: Pg 211, #16

Example 10: Pg211, #18

5-3   Binomial Probability Distributions:

Def: A **binomial probability distribution** results from a procedure that meets all the following requirements.

    a. The procedure has a fixed number of trials.
 1. The trials must be independent. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
 2. Each trial must have all outcomes classified into two categories (commonly referred to as success and failure).
 3. The probability of a success remains the same in all trials.

Notation for binomial probability distributions:

n   the number of times a trial is repeated

p =P(S)   the probability of success in a single trial

q = P(F) the probability of failure in a single trial (q = 1 - p)

x   the random variable represents a specific number of successes in n trials: x = 0, 1, 2, ..., n.

P(x)   the probability of getting exactly x successes among the n trials.

Note that success doesn't mean something good. Either of the two possible categories may be called the success. x and p both refer to the same category which is success. p is the probability of getting a success on just one trial.

Example 11: AT&T claims that when customers call directly assistance for telephone numbers, the right number is given 90% of the time. Assume a 90% rate of correct responses, and assume that we want to find the probability that among 5 requests, 3 of the responses are correct. a) Does this procedure result in a binomial distribution? b) If this procedure does result in a binomial distribution, identify the values of n, x, p, and q.

There are 3 methods for finding the probabilities corresponding to the random variable x in a binomial distribution.

I)   **Binomial formula**: In a binomial distribution, the probability of exactly s successes in n trial is $P(x) = {}^n_x C \, p^x \, q^{n-x} = \frac{n!}{(n-x)!x!} \, p^x \, q^{n-x}$

Rounding off probabilities: when expressing the value of a probability, either give the exact fraction or decimal or round off final decimal results to three significant digits.

Example 12 a): Find the probability that among 5 results, 3 of the responses are correct in example 11.

II)   **Using Table A-1**
       1st: Locate the n and the corresponding value of x that is desired.

2nd: Look at this row with the proper probability of p using the column across the top.

Example 12 b) In example 11, find the probability exactly 4 of the responses are correct.

III)   **Using Technology: 1.** Press 2nd VARS   2. Highlight binompdf, and enter.   3. With binompdf on the screen, type the number of trials, p, and x. Hit enter.

Cumulative Probability: P(X$\leq x$) The sum of all probabilities from x = 0 through the specific value entered for x.

Example 13: A survey indicates that 21% of American men consider fishing as their favorite leisure-time activity. You randomly select five men and ask them if fishing is their favorite leisure-time activity. Find the probability that a) exactly two of them respond yes, and b) fewer than two of them respond yes.

Example 14: Southwest Air had the best rate with 80% of its flights arriving on time. A test is conducted by randomly selecting 15 flights. a) Find the probability that at least 10 flights arrive on time. b) Find the probability that at least 10 flights arrive late?   c) Would it be unusual for Southwest air to have 5 flights arrive late? **Hint**: (At least 10 flights are late is the same as at most 5 flights are on time)

Try it yourself: A test consists of multiple-choice questions, each having four possible answers, one of which is correct. Assume that you guess the answers to six such questions. What is the probability of getting at least three correct answers when six guesses are made?

5-4   Mean, Variance, and standard deviation for the Binomial Distribution

Mean: $\mu = n \cdot p$ Variance: $\sigma^2 = n \cdot p \cdot q$ standard deviation: $\sigma = \sqrt{n \cdot p \cdot q}$

Example 15: According to the federal Communication commission, 70% of all U.S. households have cable television in 2002. In a simple random sample of 300 households, determine the mean and standard deviation number of households that will have cable television.

As the number of trials, n, in a binomial experiment increase, the probability distribution of the random variable x becomes bell-shaped. As a rule of thumb, if $n \cdot p \cdot q \geq 10$, the probability distribution will be approximately bell shaped.

Try it yourself: Do # 18 in section 5-4