# Review for Final Exam

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Mechanical Engineering 375

***Heat Transfer***

May 16, 2007

California State University Northridge

## Outline

- Basic equations, thermal resistance
- Heat sources
- Conduction, steady and unsteady
- Computing convection heat transfer
  - Forced convection, internal and external
  - Natural convection
- Radiation properties
- Radiative Exchange

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## Final Exam

- Wednesday, May 23, 3 – 5 pm
- Open textbook/one-page equation sheet
- Problems like homework, midterm and quiz problems
- Cumulative with emphasis on second half of course
- Complete basic approach to all problems rather than finishing details of algebra or arithmetic

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## Basic Equations

- Fourier law for heat conduction (1D)

$$\dot{q} = \frac{k(T_1 - T_2)}{L} \quad or \quad \dot{Q} = \dot{q}A = \frac{kA(T_1 - T_2)}{L}$$

- Convection heat transfer

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

- Radiation (from small object, 1, in large enclosure, 2)

$$\dot{Q}_{rad,1\to 2} = A_1\varepsilon_1\sigma\left(T_1^4 - T_2^4\right)$$

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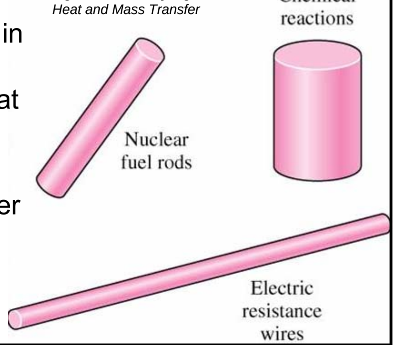
## Heat Generation

- Various phenomena in solids can generate heat
- Define $\dot{e}_{gen}$ as the heat generated per unit volume per unit time

Figure 2-21 from Çengel, *Heat and Mass Transfer*

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## Rectangular Energy Balance

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial \dot{q}_x}{\partial x} - \frac{\partial \dot{q}_y}{\partial y} - \frac{\partial \dot{q}_z}{\partial z} + \dot{e}_{gen}$$

Stored energy = heat inflow – heat outflow + heat generated

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}k\frac{\partial T}{\partial x} + \frac{\partial}{\partial y}k\frac{\partial T}{\partial y} + \frac{\partial}{\partial z}k\frac{\partial T}{\partial z} + \dot{e}_{gen}$$

Uses Fourier Law $\dot{q}_\xi = -k\frac{\partial T}{\partial \xi}$

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## Cylindrical Coordinates

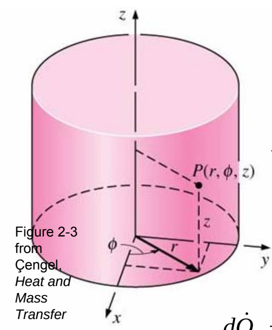


Figure 2-3 from Çengel, *Heat and Mass Transfer*

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} kr \frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial \phi} k \frac{\partial T}{\partial \phi} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + \dot{e}_{gen}$$

$$d\dot{Q}_r = \dot{q}_r dA = -k \frac{\partial T}{\partial r} r d\phi dz$$

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## Spherical Coordinates

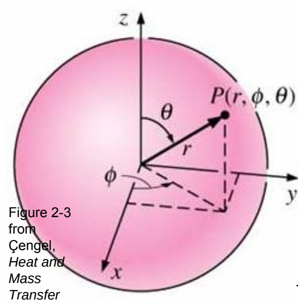


Figure 2-3 from Çengel, *Heat and Mass Transfer*

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} kr^2 \frac{\partial T}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta} k \sin\theta \frac{\partial T}{\partial \theta} + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi} k \frac{\partial T}{\partial \phi} + \dot{e}_{gen}$$

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## 1-D, Rectangular, Heat Generation

- Temperature profile for generation with $T = T_0$ at $x = 0$ and $T = T_L$ at $x = L$

$$T = T_0 - \frac{\dot{e}_{gen} x^2}{2k} + \frac{\dot{e}_{gen} xL}{2k} - \frac{(T_0 - T_L)x}{L}$$

$$\dot{q} = -k\frac{dT}{dx} = -k\left[-\frac{\dot{e}_{gen} 2x}{2k} + \frac{\dot{e}_{gen} L}{2k} - \frac{(T_0 - T_L)}{L}\right]$$

$$\dot{q} = \frac{\dot{e}_{gen}(2x - L)}{2} + \frac{k(T_0 - T_L)}{L}$$

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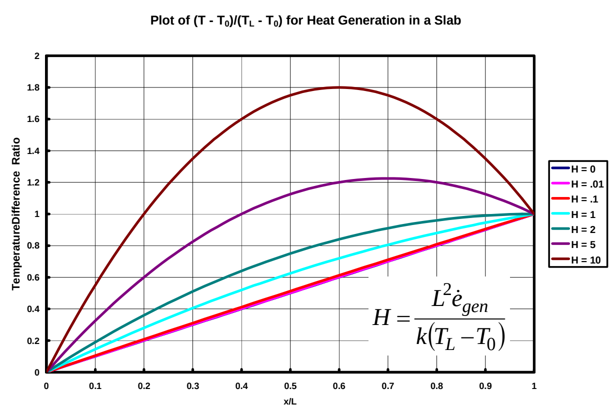


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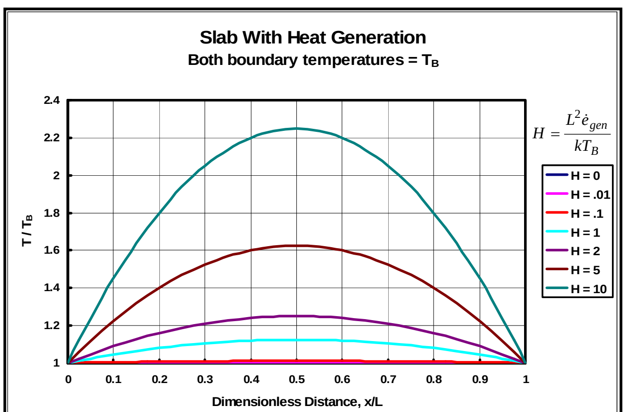


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## Thermal Resistance

- Conduction

$$\dot{Q} = \frac{\bar{k}A(T_1 - T_2)}{L} \Rightarrow \dot{Q} = \frac{T_1 - T_2}{R_{cond}} \Rightarrow R_{cond} = \frac{L}{kA}$$

- Convection

$$\dot{Q} = hA(T_s - T_f) \Rightarrow \dot{Q} = \frac{T_s - T_f}{R_{conv}} \Rightarrow R_{conv} = \frac{1}{hA}$$

- Radiation

$$R_{rad} = \frac{1}{A_1 F_{12} \sigma (T_1^3 + T_2^3 + T_2^2 T_1 + T_1^2 T_2)} = \frac{1}{A_1 h_{rad}}$$

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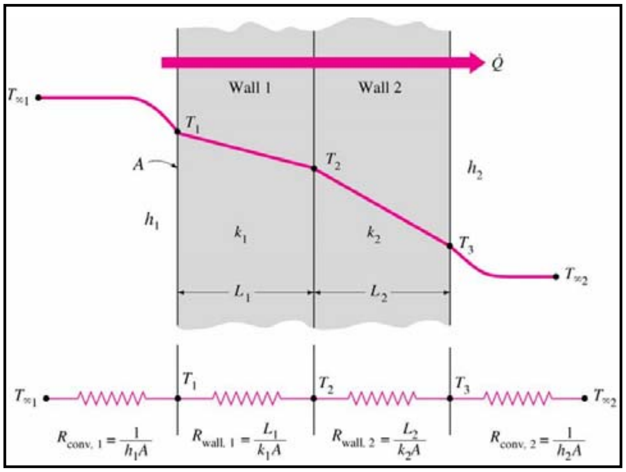
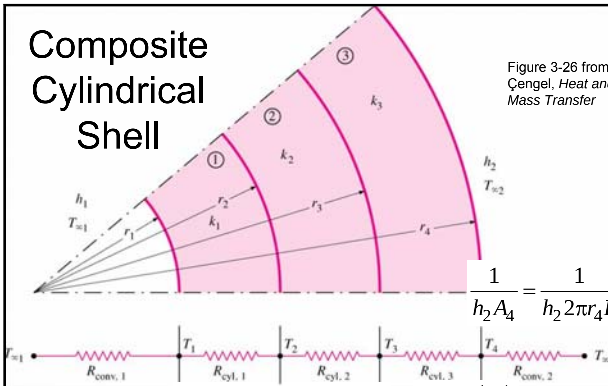



## Composite Cylindrical Shell

Figure 3-26 from Çengel, *Heat and Mass Transfer*




$$\frac{1}{h_2 A_4} = \frac{1}{h_2 2\pi r_4 L}$$

$$\frac{1}{h_1 A_1} = \frac{1}{h_1 2\pi r_1 L} \qquad \frac{1}{k_1 L}\ln\left(\frac{r_2}{r_1}\right) \quad \frac{1}{k_2 L}\ln\left(\frac{r_3}{r_2}\right) \quad \frac{1}{k_3 L}\ln\left(\frac{r_4}{r_3}\right)$$

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## Fin Results

- Infinitely long fin

$$\theta = \theta_b e^{-mx} \quad \Rightarrow \quad T - T_\infty = (T_b - T_\infty)e^{-x\sqrt{hp/kA_c}}$$

$$\dot{Q}_{x=0} = A_c \dot{q}_{x=0} = \sqrt{kA_c hp}\,(T_b - T_\infty)$$

- Heat transfer at end ($L_c = A/p$)

$$\theta = T - T_\infty = \theta_b \frac{\cosh m(L_c - x)}{\cosh mL_c} = (T_b - T_\infty)\frac{\cosh m(L_c - x)}{\cosh mL}$$

$$\dot{Q}_{x=0} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL$$

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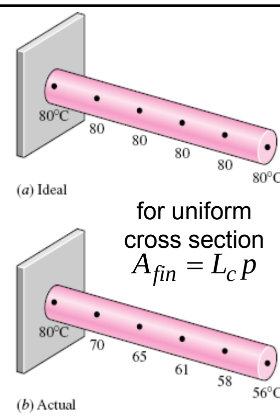
## Fin Efficiency



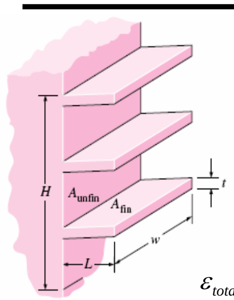

for uniform cross section $A_{fin} = L_c p$

- Compare actual heat transfer to ideal case where entire fin is at base temperature

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}} = \frac{\dot{Q}_{fin}}{hA_{fin}(T_b - T_\infty)}$$

California State University Northridge Figure 3-39 from Çengel, *Heat Transfer* 16

## Overall Fin Effectiveness




- Original area, A = (area with fins, $A_{fin}$) + (area without fins, $A_{unfin}$)

$$\frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{h(\eta_{fin}A_{fin} + A_{unfin})(T_b - T_\infty)}{hA_{no\ fin}(T_b - T_\infty)}$$

$$\varepsilon_{total} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \left(\eta_{fin}\frac{A_{fin}}{A_{no\ fin}} + \frac{A_{unfin}}{A_{no\ fin}}\right)$$

$A_{no\ fin} = w \times H$
$A_{unfin} = w \times H - 3 \times (t \times w)$
$A_{fin} = 2 \times L \times w + t \times w$
$\cong 2 \times L \times w$ (one fin)

Figure 3-45 from Çengel, *Heat Transfer* 17

## Lumped Parameter Model

- Assumes same temperature in solid
- Use characteristic length $L_c = V/A$

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$(T - T_\infty) = (T_i - T_\infty)e^{-bt} \quad or \quad T = (T_i - T_\infty)e^{-bt} + T_\infty$$

- Must have Bi = $hL_c/k < 0.1$ to use this

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## Transient 1D Convection

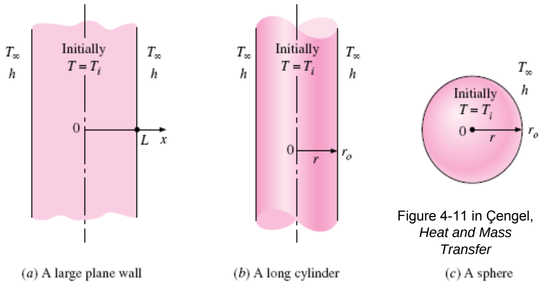


Figure 4-11 in Çengel, *Heat and Mass Transfer*

All problems have similar chart solutions

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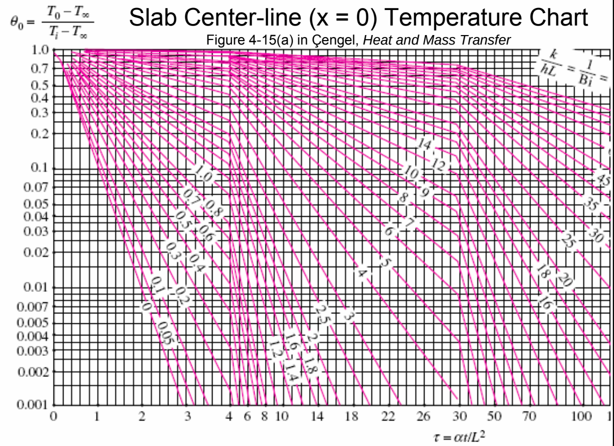
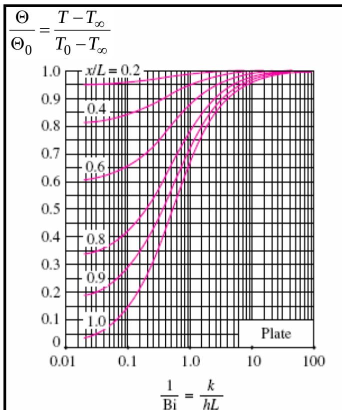
## Slab Center-line (x = 0) Temperature Chart

Figure 4-15(a) in Çengel, *Heat and Mass Transfer*




## Chart II




Can find T at any x/L from this chart once T at x = 0 is found from previous chart

See basis for this chart on the next page

Figure 4-15(b) in Çengel, *Heat and Mass Transfer*

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## Approximate Solutions

- Valid for for $\tau$ > 0.2
- Slab $\Theta = \frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1 \xi$
- Cylinder $\Theta = \frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0\left(\lambda_1 \frac{r}{r_0}\right)$
- Sphere $\Theta = \frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{r_0}{\lambda_1 r} \sin\left(\lambda_1 \frac{r}{r_0}\right)$
  - Values of $A_1$ and $\lambda_1$ depend on Bi and are different for each geometry (as is Bi)

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## Semi-Infinite Solids

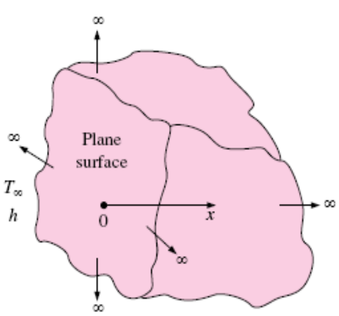


Figure 4-24 in Çengel, *Heat and Mass Transfer*

Plane that extends to infinity in all directions

Practical applications: large area for short times

- Example: earth surface locally

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## Multidimensional Solutions

- Can get multidimensional solutions as product of one dimensional solutions
  - All one-dimensional solutions have initial temperature, $T_i$, with convection coefficient, h, and environmental temperature, $T_\infty$, starting at t = 0
  - General rule: $\Theta_{twoD} = \Theta_{one}\Theta_{two}$ where $\Theta_{one}$ and $\Theta_{two}$ are solutions from charts for plane, cylinder or sphere

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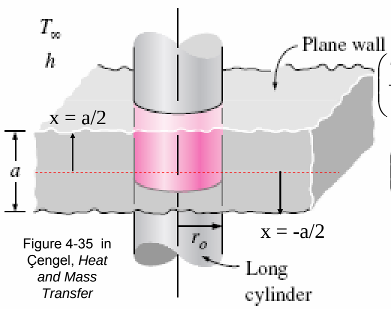
## Multidimensional Example




$$\left(\frac{T(r,x,t)-T_\infty}{T_i-T_\infty}\right)_{\text{finite cylinder}} = \left(\frac{T(r,t)-T_\infty}{T_i-T_\infty}\right)_{\text{infinite cylinder}} \bullet \left(\frac{T(x,t)-T_\infty}{T_i-T_\infty}\right)_{\text{infinite slab}}$$

Figure 4-35 in Çengel, *Heat and Mass Transfer*

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## Flow Classifications

- Forced versus free
- Internal (as in pipes) versus external (as around aircraft)
  - Entry regions in pipes *vs*. fully-developed
- Unsteady (changing with time) versus unsteady (not changing with time)
- Laminar versus turbulent
- Compressible versus incompressible
- Inviscid flow regions ($\mu$ not important)
- One-, two- or three-dimensional

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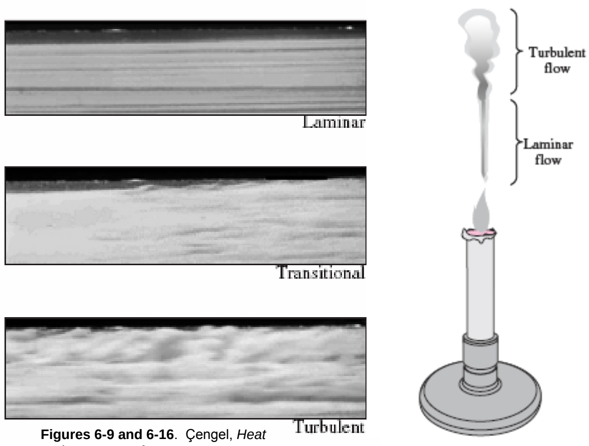
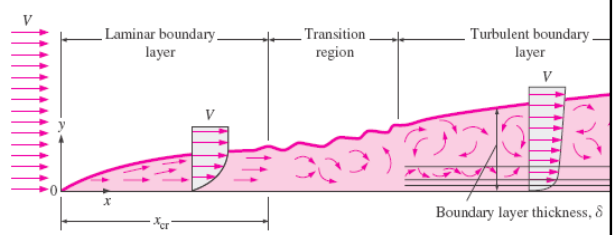



**Figures 6-9 and 6-16**. Çengel, *Heat and Mass Transfer*

## Boundary Layer

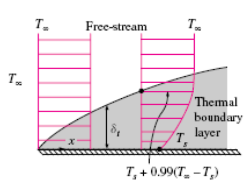



- Region near wall with sharp gradients
  - Thickness, $\delta$, usually very thin compared to overall dimension in y direction

California State University Northridge Figure 6-12 from Çengel, *Heat and Mass Transfer* 28

## Thermal Boundary Layer

- Thin region near solid surface in which most of temperature change occurs




- Thermal boundary layer thickness may be less than, greater than or equal to that of the momentum boundary layer

California State University Northridge **Figure 6-15**. Çengel, *Heat and Mass Transfer* 29

## Dimensionless Convection

- Nusselt number, $Nu = hL_c/k_{fluid}$
  - Different from $Bi = hL_c/k_{solid}$
- Reynolds number, $Re = \rho V L_c/\mu = V L_c/\nu$
- Prandtl number $Pr = \mu c_p/k$ (in tables)
- Grashof number, $Gr = \beta g \Delta T L_c^3/\nu^2$
  - g = gravity, $\beta$ = expansion coefficient = $-(1/\rho)(\partial\rho/\partial T)_p$, and $\Delta T = |T_{wall} - T_\infty|$
- Peclet, Pe = RePr; Rayleigh, Ra = GrPr

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## Characteristic Length

- Can use length as a subscript on dimensionless numbers to show correct length to use in a problem
  - $Re_D = \rho VD/\mu$, $Re_x = \rho Vx/\mu$, $Re_L = \rho VL/\mu$
  - $Nu_D = hD/k$, $Nu_x = hx/k$, $Nu_L = hL/k$
  - $Gr_D = \rho^2\beta g\Delta TD^3/\mu^2$, $Gr_x = \rho^2\beta g\Delta Tx^3/\mu^2$, $Gr_L = \rho^2\beta g\Delta TL^3/\mu^2$
- Use not necessary if meaning is clear

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## How to Compute h

- Follow this general pattern
  - Find equations for h for the description of the flow given
    - Correct flow geometry (local or average h?)
    - Free or forced convection
  - Determine if flow is laminar or turbulent
    - Different flows have different measures to determine if the flow is laminar or turbulent based on the Reynolds number, Re, for forced convection and the Grashof number, Gr, for free convection

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## How to Compute h

- Continue to follow this general pattern
  - Select correct equation for Nu (laminar or turbulent; range of Re, Pr, Gr, etc.)
  - Compute appropriate temperature for finding properties
  - Evaluate fluid properties ($\mu$, k, $\rho$, Pr) at the appropriate temperature
  - Compute Nusselt number from equation of the form $Nu = C\, Re^a\, Pr^b$ or $D\, Ra^c$
  - Compute $h = k\, Nu / L_C$

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## Property Temperature

- Find properties at correct temperature
- Some equations specify particular temperatures to be used (e.g. $\mu/\mu_w$)
- External flows and natural convection use film temperature $(T_w + T_\infty)/2$
- Internal flows use mean fluid temperature $(T_{in} + T_{out})/2$

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## Key Ideas of External Flows

- The flow is unconfined
- Moving objects into still air are modeled as still objects with air flowing over them
- There is an approach condition of velocity, $U_\infty$, and temperature, $T_\infty$
- Far from the body the velocity and temperature remain at $U_\infty$ and $T_\infty$
- $T_\infty$ is the (constant) fluid temperature used to compute heat transfer

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## Flat Plate Flow Equations

- Laminar flow ($Re_x$, $Re_L$ < 500,000, Pr > .6)

$$C_{f_x} = \frac{\tau_{wall}}{\rho U_\infty^2/2} = 0.664\,\mathrm{Re}_x^{-1/2} \qquad Nu_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\overline{C}_f = \frac{\overline{\tau}_{wall}}{\rho U_\infty^2/2} = 1.33\,\mathrm{Re}_L^{-1/2} \qquad \overline{Nu}_L = \frac{\overline{h}L}{k} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

- Turbulent flow ($5\times10^5 < Re_x, Re_L < 10^7$)

$$C_{f_x} = \frac{\tau_{wall}}{\rho U_\infty^2/2} = 0.059\,\mathrm{Re}_x^{-1/5} \qquad Nu_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

$$\overline{C}_f = \frac{\overline{\tau}_{wall}}{\rho U_\infty^2/2} = 0.074\,\mathrm{Re}_L^{-1/5} \qquad \overline{Nu}_L = \frac{\overline{h}L}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$$

For turbulent Nu, .6 < Pr < 60

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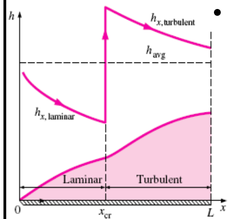
## Flat Plate Flow Equations II




- Average properties for combined laminar and turbulent regions with transition at $x_c$ = 500000 ν/U∞
  - Valid for $5\times10^5 < Re_L < 10^7$ and $0.6 < Pr < 60$

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad Nu_L = \frac{\bar{h}L}{k} = \left(0.037\,Re_L^{0.8} - 871\right)Pr^{1/3}$$

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Figure 7-10 from Çengel, *Heat and Mass Transfer*

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## Heat Transfer Coefficients

- Cylinder average h (RePr > 0.2; properties at $(T_\infty + T_s)/2$

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,Re^{1/2}\,Pr^{1/2}}{\left[1+\left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5}$$

- Sphere average h ($3.5 \le Re \le 80{,}000$; $0.7 \le Pr \le 380$; $\mu_s$ at $T_s$; other properties at $T_\infty$)

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,Re^{1/2} + 0.06\,Re^{2/3}\right]Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

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## Other Shapes and Equations

| Cross-section of the cylinder | Fluid | Range of Re | Nusselt number |
|---|---|---|---|
| Circle (D) | Gas or liquid | 0.4–4<br>4–40<br>40–4000<br>4000–40,000<br>40,000–400,000 | $Nu = 0.989Re^{0.330}\,Pr^{1/3}$<br>$Nu = 0.911Re^{0.385}\,Pr^{1/3}$<br>$Nu = 0.683Re^{0.466}\,Pr^{1/3}$<br>$Nu = 0.193Re^{0.618}\,Pr^{1/3}$<br>$Nu = 0.027Re^{0.805}\,Pr^{1/3}$ |
| Square (D) | Gas | 5000–100,000 | $Nu = 0.102Re^{0.675}\,Pr^{1/3}$ |
| Square (tilted 45°) (D) | Gas | 5000–100,000 | $Nu = 0.246Re^{0.588}\,Pr^{1/3}$ |

Part of Table 7-1 from Çengel, *Heat and Mass Transfer*

## Tube Bank Heat Transfer

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, 1987)*

| Arrangement | Range of $Re_D$ | Correlation |
|---|---|---|
| In-line | 0–100 | $Nu_D = 0.9\,Re_D^{0.4}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |
| | 100–1000 | $Nu_D = 0.52\,Re_D^{0.5}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |
| | $1000–2\times10^5$ | $Nu_D = 0.27\,Re_D^{0.63}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |
| | $2\times10^5–2\times10^6$ | $Nu_D = 0.033\,Re_D^{0.8}Pr^{0.4}(Pr/Pr_s)^{0.25}$ |
| Staggered | 0–500 | $Nu_D = 1.04\,Re_D^{0.4}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |
| | 500–1000 | $Nu_D = 0.71\,Re_D^{0.5}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |
| | $1000–2\times10^5$ | $Nu_D = 0.35(S_T/S_L)^{0.2}\,Re_D^{0.6}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |
| | $2\times10^5–2\times10^6$ | $Nu_D = 0.031(S_T/S_L)^{0.2}\,Re_D^{0.8}Pr^{0.36}(Pr/Pr_s)^{0.25}$ |

*All properties except $Pr_s$ are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid ($Pr_s$ is to be evaluated at $T_s$).

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Table 7-2 from Çengel, *Heat and Mass Transfer*

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## Key Ideas of Internal Flows

- The flow is confined
- There is a temperature and velocity profile in the flow
  - Use average velocity and temperature
- Wall fluid heat exchange will change the average fluid temperature
  - There is no longer a constant fluid temperature like $T_\infty$ for computing heat transfer

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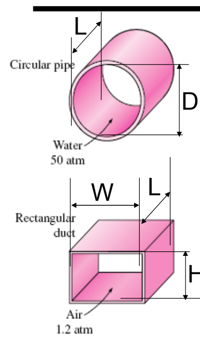
## Area Terms




- $A_{cs}$ is cross-sectional area for the flow
  - $A_{cs} = \pi D^2/4$ for circular pipe
  - $A_{cs}$ = WH for rectangular duct
- $A_w$ is the wall area for heat transfer
  - $A_w = \pi DL$ for circular pipe
  - $A_w = 2(W + H)L$ for rectangular duct

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Figure 8-1 from Çengel, *Heat and Mass Transfer*

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## Average Temperature Change

- Let T represent the average fluid temperature (instead of $T_{avg}$, $T_m$ or $\overline{T}$)
- T will change from inlet to outlet of confined flow
  - This gives a variable driving force ($T_{wall}$ – $T_{fluid}$) for heat transfer
  - Can accommodate this by using the first law of thermodynamics: $\dot{Q} = \dot{m}c_p(T_{out} - T_{in})$
  - Two cases: fixed wall heat flux and fixed wall temperature

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## Fixed Wall Heat Flux

- Fixed wall heat flux, $\dot{q}_{wall}$, over given wall area, $A_w$, gives total heat input which is related to $T_{out} - T_{in}$ by thermodynamics

$$\dot{Q} = \dot{q}_{wall} A_w = \dot{m}c_p\left(T_{out} - T_{in}\right) \quad \Rightarrow \quad T_{out} = T_{in} + \frac{\dot{q}_{wall} A_w}{\dot{m}c_p}$$

- "Outlet" can be any point along flow path where area from inlet is $A_w$
- We can compute $T_w$ at this point as $T_w = T_{out} + \dot{q}_{wall}/h$

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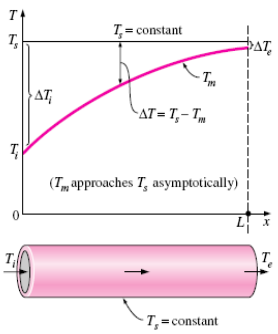
## Constant Wall Temperature




$$\left(T_{out} - T_s\right) = \left(T_{in} - T_s\right) e^{-\frac{hA_w}{\dot{m}c_p}}$$

- $hA_w / \dot{m}c_p$ = NTU, the number of transfer units
- This is general equation for computing $T_{out}$ in internal flows

California State University Northridge Figure 8-14 from Çengel, *Heat and Mass Transfer* 45

## Log-mean Temperature Diff

- This is usually written as a set of temperature differences

$$LM\Delta T = \frac{\left(T_{out} - T_{in}\right)}{\ln\left(\dfrac{T_{out} - T_s}{T_{in} - T_s}\right)} = \frac{\left(T_{out} - T_s\right) - \left(T_{in} - T_s\right)}{\ln\left(\dfrac{T_{out} - T_s}{T_{in} - T_s}\right)}$$

$$\dot{Q} = \frac{hA_w\left(T_{out} - T_{in}\right)}{\ln\left(\dfrac{T_{out} - T_s}{T_{in} - T_s}\right)} = hA_w\left(LM\Delta T\right)$$

Çengel uses $\Delta T_{lm}$ for LMΔT

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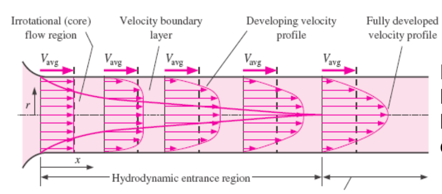
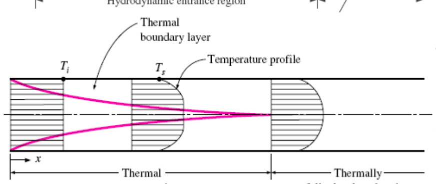
## Developing Flows




Momentum boundary layer development




Thermal boundary layer development

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## Fully Developed Flow

- Temperature profile does not change with x if flow is fully developed thermally
- This means that ∂T/∂r does not change with downstream distance, x, so heat flux (and Nu) do not depend on x
- Laminar entry lengths $\dfrac{L_h}{D} \approx 0.05\,\mathrm{Re} \qquad \dfrac{L_t}{D} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}$
- Turbulent entry lengths $\dfrac{L_t}{D} \approx \dfrac{L_h}{D} = 1.359\,\mathrm{Re}^{1/4} \approx 10$

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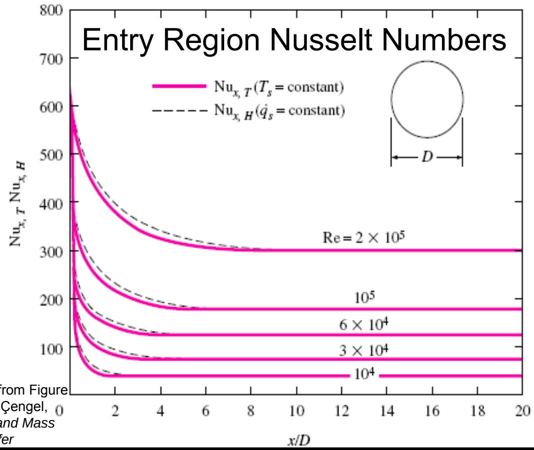
## Entry Region Nusselt Numbers




Eggs from Figure 8-9 in Çengel, *Heat and Mass Transfer*

## Internal Flow Pressure Drop

- General formula: $\Delta p = f\,(L/D)\,\rho V^2/2$
- Friction factor, f, depends on Re = $\rho VD/\mu$ and relative roughness, $\varepsilon/D$
- For laminar flows, f = 64/Re
  - No dependence on relative roughness
- For turbulent flows

Colebrook $$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Haaland $$\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left(\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$$

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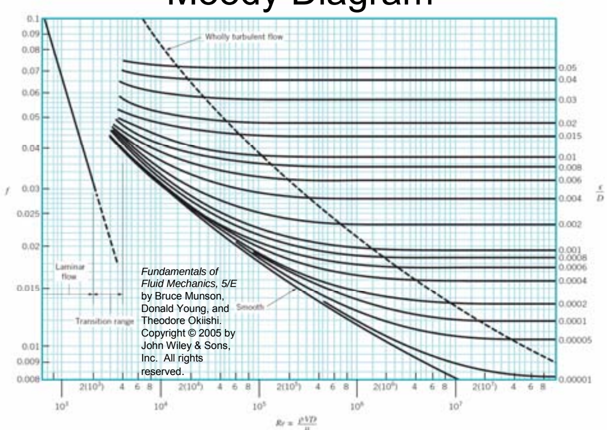
## Moody Diagram




*Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi. Copyright © 2005 by John Wiley & Sons, Inc. All rights reserved.

## Laminar Nusselt Number

- Laminar flow if Re = $\rho VD/\mu$ < 2,300
- Fully-developed, constant heat flux, Nu = 4.36
- Fully-developed, constant wall temperature: Nu = 3.66
- Entry region, constant wall temperature:

$$Nu = 3.66 + \frac{0.065(D/L)\mathrm{Re}\,\mathrm{Pr}}{1 + 0.04\left[(D/L)\mathrm{Re}\,\mathrm{Pr}\right]^{2/3}}$$

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## Noncircular Ducts

- Define hydraulic diameter, $D_h = 4A/P$
  - A is cross-sectional area for flow
  - P is wetted perimeter
  - For a circular pipe where A = pD2/4 and P = $\pi$D, $D_h = 4(\pi D^2/4) / (\pi D) = D$
- For turbulent flows use Moody diagram with D replaced by $D_h$ in Re, f, and $\varepsilon/D$
- For laminar flows, f = A/Re and Nu = B (all based on $D_h$) – A and B next slide

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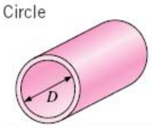
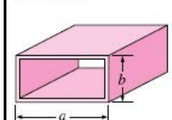
**TABLE 8–1**

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, Re = $V_{avg}D_h/\nu$, and Nu = $hD_h/k$)

| Tube Geometry | a/b or θ° | Nusselt Number $T_s$ = Const. | Nusselt Number $\dot{q}_s$ = Const. | Friction Factor f |
|---|---|---|---|---|
| Circle  | — | 3.66 | 4.36 | 64.00/Re |
| Rectangle  | a/b | | | |
| | 1 | 2.98 | 3.61 | 56.92/Re |
| | 2 | 3.39 | 4.12 | 62.20/Re |
| | 3 | 3.96 | 4.79 | 68.36/Re |
| | 4 | 4.44 | 5.33 | 72.92/Re |
| | 6 | 5.14 | 6.05 | 78.80/Re |
| | 8 | 5.60 | 6.49 | 82.32/Re |
| | ∞ | 7.54 | 8.24 | 96.00/Re |

From Çengel, *Heat and Mass Transfer*

## Turbulent Flow

- Smooth tubes (Gnielinski)

$$Nu = \frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(f/8)^{0.5}\left(\mathrm{Pr}^{2/3}-1\right)} \quad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3x10^3 < \mathrm{Re} < 5x10^6 \end{pmatrix}$$

Petukhov : $f = [0.790\ln(\mathrm{Re})-1.64]^{-2} \qquad 3000 < \mathrm{Re} < 5x10^6$

- Tubes with roughness
  - Use correlations developed for this case
  - As approximation use Gnielinski equation with f from Moody diagram or f equation
    - Danger! h does not increase for f >4$f_{smooth}$

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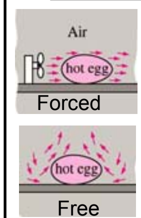
## Free (Natural) Convection



(Natural)

Eggs from Figure 1-33 in Çengel, *Heat and Mass Transfer*

- Flow is induced by temperature difference
  - No external source of fluid motion
  - Temperature differences cause density differences
  - Density differences induce flow
    - "Warm air rises"
  - Volume expansion coefficient: β = [–(1/ρ)(∂ρ/∂T)]
    - For ideal gases β = 1/T

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## Grashof and Rayleigh Numbers

- Dimensionless groups for free (natural) convection

$$Gr = \frac{\beta g \Delta T L_c^3}{\nu^2} = \frac{\rho^2 \beta g \Delta T L_c^3}{\mu^2} \qquad Ra = Gr\,\mathrm{Pr} = \frac{\beta g \Delta T L_c^3}{\nu\alpha}$$

- g = acceleration of gravity ($LT^{-2}$)
- β = –(1/ρ)(∂ρ/∂T) called the volume expansion coefficient (dimensions: 1/Θ)
- ΔT = |$T_{wall}$ – $T_{fluid}$| (dimensions: Θ)
- Other terms same as previous use

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## Equations for Nu

- Equations have form of $AGr^aPr^b$ or $BRa^c$
- Since Gr and Ra contain |$T_{wall}$ – $T_{fluid}$|, an iterative process is required if one of these temperatures is unknown
- Transition from laminar to turbulent occurs at given Ra values
  - For vertical plate transition Ra = $10^9$
- Evaluate properties at "film" (average) temperature, ($T_{wall}$ + $T_{fluid}$)/2

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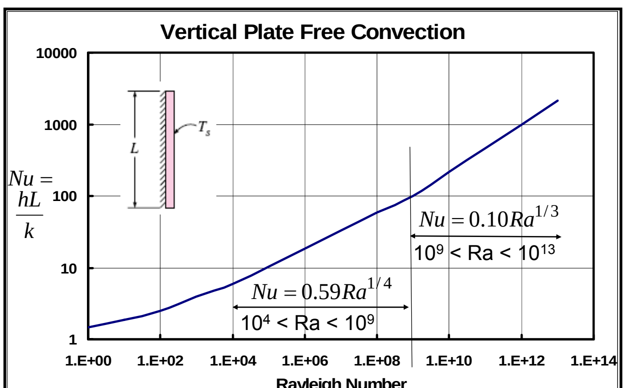


Plate figure from Table 9-1 in Çengel, *Heat and Mass Transfer*

$$Ra = \frac{\beta g \Delta T L^3}{\nu^2}\mathrm{Pr} = \frac{\beta g \Delta T L^3}{\nu\alpha}$$

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## Vertical Plate Free Convection

- Simplified equations on previous chart for constant wall temperature
  - More accurate: Churchill and Chu, any Ra

$$Nu_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+(0.492/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2 \qquad Any\ Ra_L$$

  - More accurate laminar Churchill/Chu

$$Nu_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/\mathrm{Pr})^{9/16}\right]^{4/9}} \qquad 0 < Ra_L < 10^9$$

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## Vertical Plate Free Convection

- Constant wall heat flux
  - Use $\dot{q} = hA(T_w - T_\infty)$ to compute an unknown temperature ($T_w$ or $T_\infty$) from known wall heat flux and computed h
  - $T_w$ varies along wall, but the average heat transfer uses midpoint temperature, $T_{L/2}$

$$\dot{q}_{wall} = hA_{wall}(T_{L/2} - T_\infty) \quad \Rightarrow \quad T_{L/2} - T_\infty = \frac{\dot{q}_{wall}}{hA_{wall}}$$

  - Use trial and error solution with $T_{L/2} - T_\infty$ as $\Delta T$ in Ra used to compute h = kNu/L

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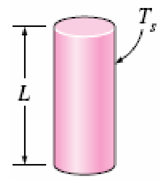
## Vertical Cylinder




Cylinder figure from Table 9-1 in Çengel, *Heat and Mass Transfer*

- Apply equations for vertical plate from previous charts if $D/L \geq 35/Gr^{1/4}$
- For this D/L effects of curvature are not important
- Thin cylinder results of Cebeci and Minkowcyz and Sparrow available in ASME Transactions

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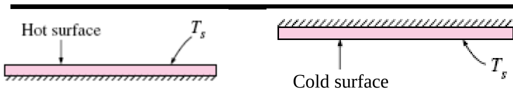
## Horizontal Plate




- Hot surface facing up or cold surface facing down
- $L_c$ = area / perimeter ($A_s$/p)
  - For a rectangle of length, L, and width, W, $L_c$ = (LW) / (2L + 2W) = 1 / ( 2 / W + 2 / L)
  - For a circle, $L_c = \pi R^2 / 2\pi R = R/2 = D/4$

Figures from Table 9-1 in Çengel, *Heat and Mass Transfer*

$$Nu = 0.54 Ra_{L_c}^{1/4} \quad 10^4 < Ra < 10^7$$

$$Nu = 0.15 Ra_{L_c}^{1/3} \quad 10^7 < Ra < 10^{11}$$

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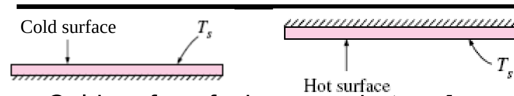
## Horizontal Plate II




- Cold surface facing up or hot surface facing down
- $L_c$ = area / perimeter ($A_s$/p)
  - For a rectangle of length, L, and width, W, $L_c$ = (LW) / (2L + 2W) = 1 / ( 2 / W + 2 / L)
  - For a circle, $L_c = \pi R^2 / 2\pi R = R/2 = D/4$

Figures from Table 9-1 in Çengel, *Heat and Mass Transfer*

$$Nu = 0.27 Ra_{L_c}^{1/4} \quad 10^5 < Ra < 10^{11}$$

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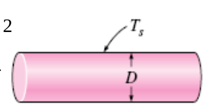
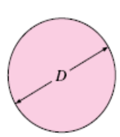
## Sphere and Horizontal Cylinder

$$Nu_D = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{8/27}} \right\}^2$$



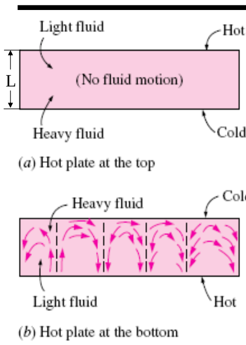

- $Nu_D$ results are average values




$$Nu_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/\mathrm{Pr})^{9/16}\right]^{4/9}}$$

California State University Northridge Figures from Çengel, *Heat and Mass Transfer* 65

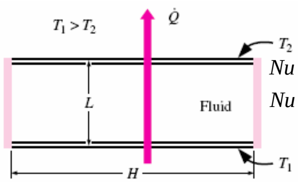
## Horizontal Enclosures




- **Top side warmer:** no convection
- Conduction only, Nu = hL/k = 1
- **Bottom warmer:** convection becomes significant when $Ra_L = (\mathrm{Pr})\beta g \Delta T L^3/\nu^2 = \beta g \Delta T L^3/\nu\alpha > 1708$

California State University Northridge Figure 9-22 in Çengel, *Heat and Mass Transfer* 66

## Horizontal Enclosures II




Jakob, for 0.5 < Pr < 2

$Nu = 0.195 Ra_L^{1/4}$ $\quad 10^4 < Ra_L < 4x10^5$

$Nu = 0.068 Ra_L^{1/3}$ $\quad 4x10^5 < Ra_L < 10^7$

Globe and Dropkin for a range of liquids

$$Nu = 0.069 Ra_L^{1/3} \Pr^{0.074} \quad 3x10^5 < Ra_L < 7x10^9$$

Hollands *et al*. for air; also for other fluids if $Ra_L < 10_5$

$$Nu = 1 + 1.44 \max\left(0, 1 - \frac{1708}{Ra_L}\right) + \max\left(0, \frac{Ra_L}{18} - 1\right) \quad Ra_L < 10^8$$



## Vertical Enclosures

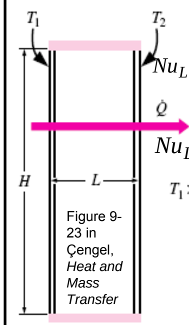



Figure 9-23 in Çengel, *Heat and Mass Transfer*

Berkovsky and Polevikov, any Pr

$$Nu_L = 0.18\left(\frac{\Pr}{0.2+\Pr} Ra_L\right)^{0.29} \quad 1 < H/L < 2, \quad Ra_L \Pr/0.2+\Pr > 10^3$$

$$Nu_L = 0.22\left(\frac{\Pr Ra_L}{0.2+\Pr}\right)^{0.28}\left(\frac{L}{H}\right)^{1/4} \quad 2 < H/L < 10, \quad Ra_L < 10^{10}$$

MacGregor and Emery

$$Nu_L = 0.42 Ra_L^{1/4} \Pr^{0.012}\left(\frac{L}{H}\right)^{0.3} \quad 10 < H/L < 40, \quad 1 < \Pr < 2x10^4, \quad 10^4 < Ra_L < 10^7$$

$1 < H/L < 40$, $1 < \Pr < 20$, $10^6 < Ra_L < 10^9$ $\quad Nu_L = 0.46 Ra_L^{1/3}$



## Heat Exchangers

- Used to transfer energy from one fluid to another
- One fluid, the hot fluid, is cooled while the other, the cold fluid, is heated
- May have phase change: temperature of one or both fluids is constant
- Simplest is double pipe heat exchanger
  - Parallel flow and counter flow



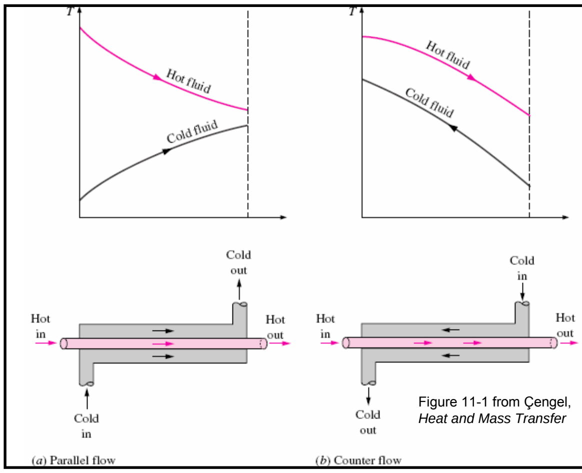



Figure 11-1 from Çengel, *Heat and Mass Transfer*

## Compact Heat Exchangers

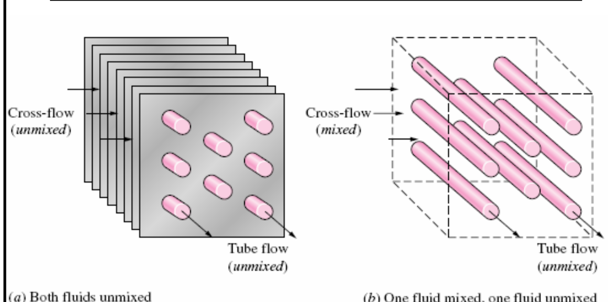
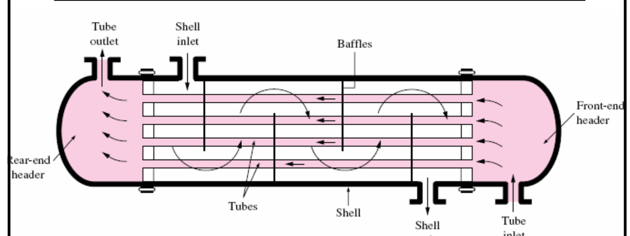



Figure 11-3 from Çengel, *Heat and Mass Transfer*



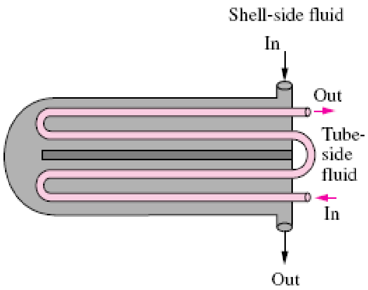
## Shell-and-Tube Exchanger




- Counter flow exchanger with larger surface area; baffles promote mixing

Figure 11-4 from Çengel, *Heat and Mass Transfer*

## Shell and Tube Passes



(b) Two-shell passes and four-tube passes

Tube flow has three complete changes of direction giving four tube passes

Shell flow changes direction to give two shell passes

Figure 11-5(b) from Çengel, *Heat and Mass Transfer*

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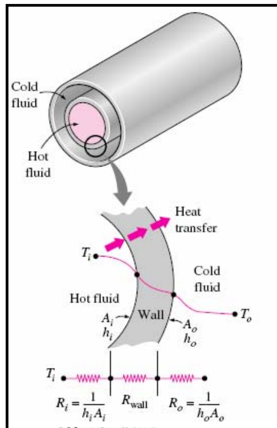
## Overall U




U is overall heat transfer coefficient

Analyzed here for double-pipe heat exchanger

$$R = \frac{1}{h_i A_i} + R_{wall} + \frac{1}{h_o A_o} = \frac{1}{U_o A_o} = \frac{1}{U_i A_i} = \frac{1}{UA}$$

Figure 11-7 from Çengel, *Heat and Mass Transfer*

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## Heat Exchange Analysis

- Heat transfer from hot to cold fluid $\dot{Q} = UA\Delta T$
- First law energy balances
$$\dot{Q} = \dot{m}_c c_{p_c}\left(T_{c,out} - T_{c,in}\right)$$
$$\dot{Q} = \dot{m}_h c_{p_h}\left(T_{h,in} - T_{h,out}\right)$$
- Assumes no heat loss to surroundings
  - Subscripts c and h denote cold and hot fluids, respectively
  - Alternative analysis for phase change

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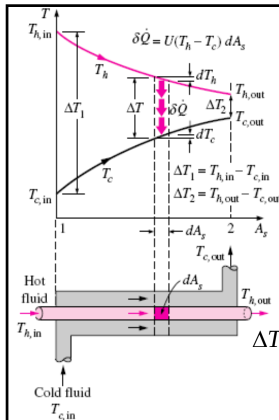
## Parallel Flow




Parallel flow heat exchanger $\dot{Q} = UA\Delta T_{lm}$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$\Delta T_{lm} = \frac{\left(T_{h,out} - T_{c,out}\right) - \left(T_{h,in} - T_{c,in}\right)}{\ln\left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}}\right)}$$

Figure 11-14 from Çengel, *Heat and Mass Transfer*

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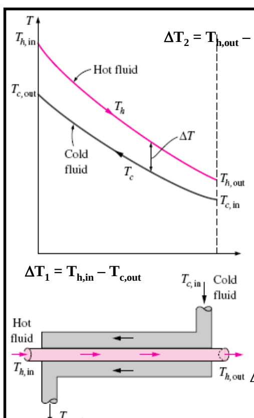
## Counter Flow




Same basic equations
- Difference in $\Delta T_1$ and $\Delta T_2$ definitions

$$\dot{Q} = UA\Delta T_{lm} = UA\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

$$\Delta T_{lm} = \frac{\left(T_{h,out} - T_{c,in}\right) - \left(T_{h,in} - T_{c,out}\right)}{\ln\left(\frac{T_{h,out} - T_{c,in}}{T_{h,in} - T_{c,out}}\right)}$$

Figure 11-16 from Çengel, *Heat and Mass Transfer*

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## Heat Exchanger Problems

- With $\Delta T_{lm}$ method we want to find U or A when all temperatures are known
- If we know three temperatures, we can find the fourth by an energy balance with known mass flow rates (and $c_p$'s)

$$\dot{Q} = \dot{m}_c c_{p_c}\left(T_{c,out} - T_{c,in}\right)$$
$$\dot{Q} = \dot{m}_h c_{p_h}\left(T_{h,in} - T_{h,out}\right)$$

Can find $\dot{Q}$ from two temperatures for one stream and then find unknown temperature

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## Correction Factors

- Correction factor parameters, R and P
  - Shell and tube definitions below

$$P = \frac{T_{tube,out} - T_{tube,in}}{T_{shell,in} - T_{tube,in}} = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_{shell,in} - T_{shell,out}}{T_{tube,out} - T_{tube,in}} = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{tube}}{(\dot{m}c_p)_{shell}}$$

  - Correction factor charts show diagrams that illustrate the equations for P and R

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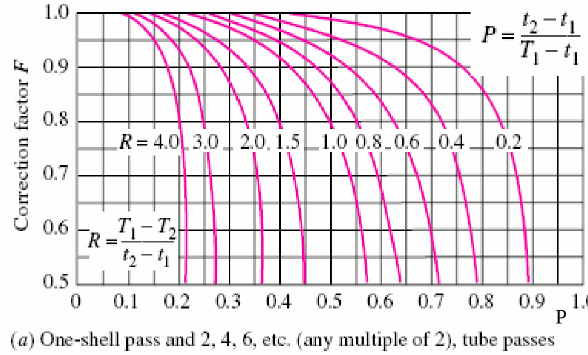


(a) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

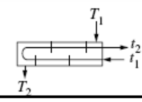


Figure 11-18 from Çengel, *Heat and Mass Transfer*

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## Effectiveness-NTU Method

- Used when not all temperatures are known
- Based on ratio of actual heat transfer to maximum possible heat transfer
- Maximum possible temperature difference, $\Delta T_{max}$ is $T_{h,in} - T_{c,in}$
  - Only one fluid, the one with the smaller value of $\dot{m}c_p$, can have $\Delta T_{max}$
  - Define $C_c = (\dot{m}c_p)_c$ and $C_h = (\dot{m}c_p)_h$

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## Effectiveness, ε

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\dot{Q}}{C_{min}(T_{h,in} - T_{c,in})} \qquad C_{min} = \min(C_h, C_c)$$

- In effectiveness-NTU method we find ε, then find $\dot{Q} = \varepsilon\dot{Q}_{max}$
  - Use $C_{min}\Delta T_{max}$ to find $\dot{Q}_{max}$ because $C_1\Delta T_1 = C_2\Delta T_2$ or $\Delta T_2 = C_1\Delta T_1/C_2$
  - If $\Delta T_2 = \Delta T_{max}$ and $C_1/C_2 > 1$, $\Delta T_2 > \Delta T_{max}$
  - $C_{min}\Delta T_{max}$ is maximum heat transfer that can occur without impossible $T < T_{c,in}$

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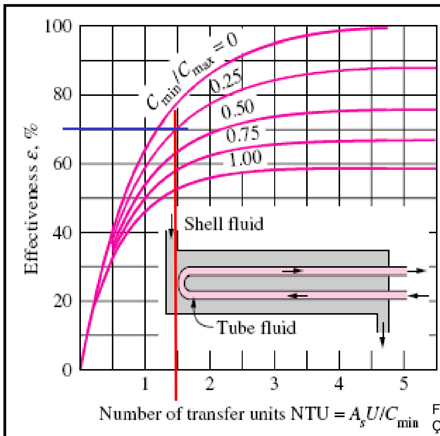


(c) One-shell pass and 2, 4, 6, … tube passes

## Find ε

Example chart for finding effectiveness from NTU = $UA/C_{min}$ and $C_{min}/C_{max}$ ratio

For NTU = 1.5 and $C_{min}/C_{max}$ = 0.25, ε = .7

Figure 11-26 from Çengel, *Heat and Mass Transfer*

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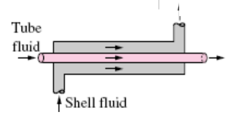
## Effectiveness Equations

- Double pipe parallel flow

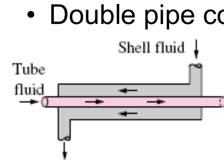
$$NTU = \frac{UA}{C_{min}}$$




$$\varepsilon = \frac{1 - e^{-NTU(1+c)}}{1 + c}$$

- Double pipe counter flow

$$c = \frac{C_{min}}{C_{max}}$$




$$\varepsilon = \frac{1 - e^{-NTU(1-c)}}{1 - ce^{-NTU(1-c)}}$$

California State University Northridge Figures from Figure 11-26 from Çengel, *Heat and Mass Transfer* 84

## Black-Body Radiation

- Basic black body equation: $E_b = \sigma T^4$
  - $E_b$ is total black-body radiation energy flux W/m² or Btu/hr·ft²
  - $\sigma$ is the Stefan-Boltzmann constant
    - $\sigma = 5.670 \times 10^{-8}$ W/m²·K⁴
    - $\sigma = 0.1714 \times 10^{-8}$ Btu/hr·ft²·R⁴
  - Must use absolute temperature
- Radiation flux varies with wavelength
  - $E_{b\lambda}$ is flux at given wavelength, $\lambda$

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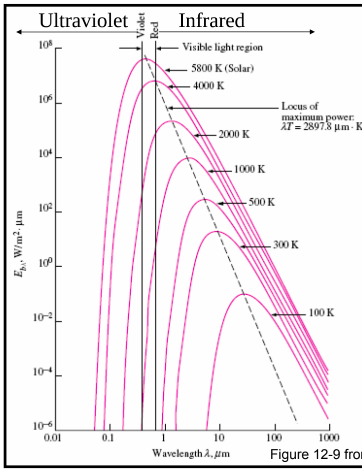
## Spectral $E_{b\lambda}$




- Energy (W/m²) emitted varies with wavelength and temperature
- Maximum point occurs where $\lambda T = 2897.8$ μm·K
- T increase shifts peak shift to lower $\lambda$

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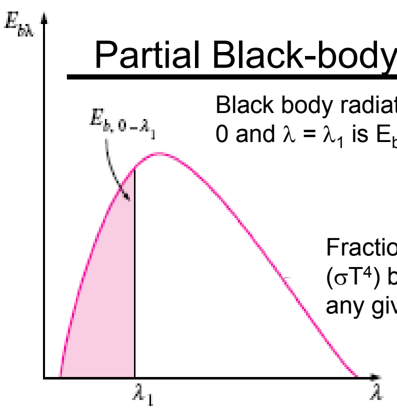
Figure 12-9 from Çengel, *Heat and Mass Transfer*

## Partial Black-body Power




Black body radiation between $\lambda = 0$ and $\lambda = \lambda_1$ is $E_{b,0\text{-}\lambda 1}$

$$E_{b,0-\lambda_1} = \int_0^{\lambda_1} E_{b\lambda} d\lambda$$

Fraction of total radiation ($\sigma T^4$) between $\lambda = 0$ and any given $\lambda$ is $f_\lambda$

$$f_\lambda = \frac{1}{\sigma T^4} \int_0^{\lambda} E_{b\lambda} d\lambda'$$

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Figure 12-13 from Çengel, *Heat and Mass Transfer*

## Radiation Tables

- Can show that $f_\lambda$ is function of $\lambda T$

$$f_\lambda = \frac{1}{\sigma T^4} \int_0^{\lambda} E_{b\lambda} d\lambda = \frac{1}{\sigma T^4} \int_0^{\lambda} \frac{C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)} d\lambda = \frac{1}{\sigma} \int_0^{\lambda T} \frac{C_1}{(\lambda T)^5 \left(e^{C_2/\lambda T} - 1\right)} d(\lambda T)$$

- Radiation tables give $f_\lambda$ versus $\lambda T$
  - See table 12-2, page 672 in text
  - Extract from this table shown at right

Blackbody radiation functions $f_\lambda$

| $\lambda T$, μm · K | $f_\lambda$ |
|---|---|
| 200 | 0.000000 |
| 400 | 0.000000 |
| 600 | 0.000000 |
| 800 | 0.000016 |
| 1000 | 0.000321 |
| 1200 | 0.002134 |
| 1400 | 0.007790 |
| 1600 | 0.019718 |
| 1800 | 0.039341 |
| 2000 | 0.066728 |

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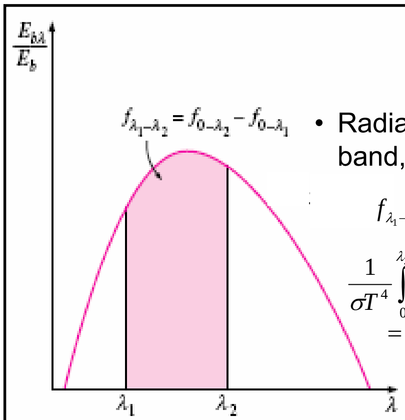
## $\Delta\lambda$




- Radiation in finite band, $\Delta\lambda$

$$f_{\lambda_1-\lambda_2} = \frac{1}{\sigma T^4} \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda = \frac{1}{\sigma T^4} \int_0^{\lambda_2} E_{b\lambda} d\lambda - \frac{1}{\sigma T^4} \int_0^{\lambda_1} E_{b\lambda} d\lambda = f_\lambda(\lambda_2 T) - f_\lambda(\lambda_1 T)$$

$$f_\lambda(0) = 0 \qquad f_\lambda(\infty) = 1$$

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Figure 12-14 from Çengel, *Heat and Mass Transfer*

## Emissivity

- Ratio of actual emissive power to blakc body emissive power
  - Diffuse surface – emissivity does not depend on direction
  - Gray surface – emissivity does not depend on wavelength
  - Gray, diffuse surface – emissivity is the does not depend on direction or wavelength
    - Simplest surface to handle and often used in radiation calculations

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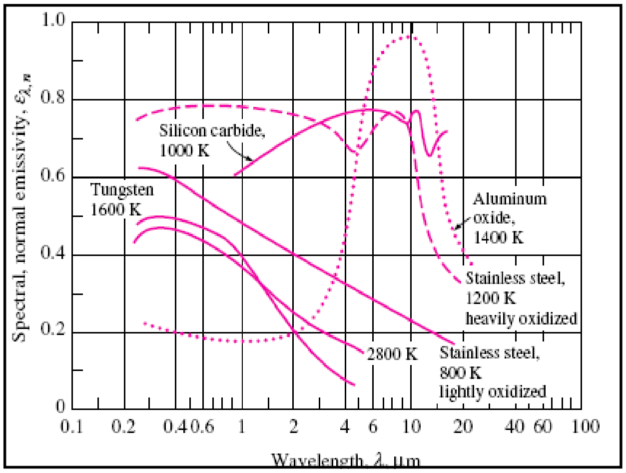


## Average Emissivity

- Average over all wavelengths

$$\varepsilon = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda = \frac{1}{\sigma T^4}\int_0^\infty \frac{\varepsilon_\lambda C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)} d\lambda = \frac{1}{\sigma}\int_0^\infty \frac{\varepsilon_\lambda C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)} d(\lambda T)$$

- For emissivity with constant values in a series of wavelength ranges

$$\varepsilon = \frac{1}{\sigma}\int_0^{\lambda_1 T} \frac{\varepsilon_1 C_1 d(\lambda T)}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)} + \frac{1}{\sigma}\int_{\lambda_1 T}^{\lambda_2 T} \frac{\varepsilon_2 C_1 d(\lambda T)}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)} + \frac{1}{\sigma}\int_{\lambda_2 T}^{\infty} \frac{\varepsilon_3 C_1 d(\lambda T)}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)}$$

$$\varepsilon = \varepsilon_1\left[f_\lambda(\lambda_1 T) - 0\right] + \varepsilon_2\left[f_\lambda(\lambda_2 T) - f_\lambda(\lambda_1 T)\right] + \varepsilon_3\left[1 - f_\lambda(\lambda_2 T)\right]$$

- Applies to other properties as well

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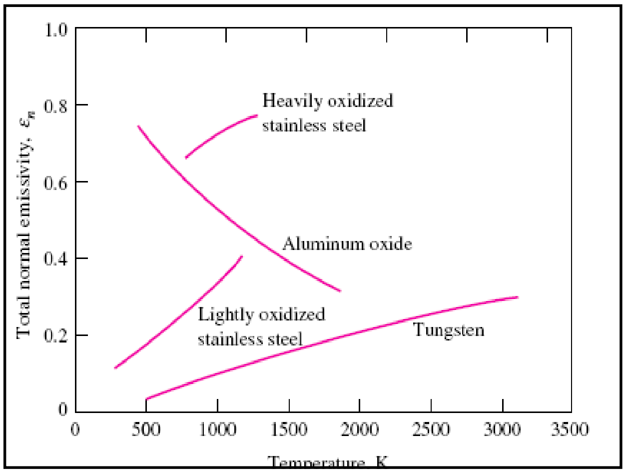


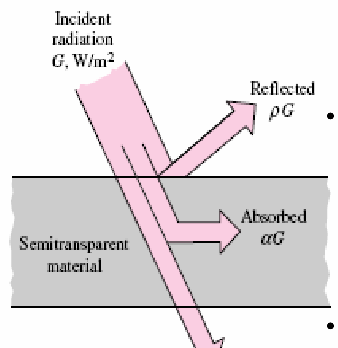


Figure 12-31 from Çengel, *Heat and Mass Transfer*

## Properties

- Incoming radiation properties
  - Reflectivity, ρ
  - Absorptivity, α
  - Transmissivity, τ
- Energy balance: $\rho + \alpha + \tau = 1$

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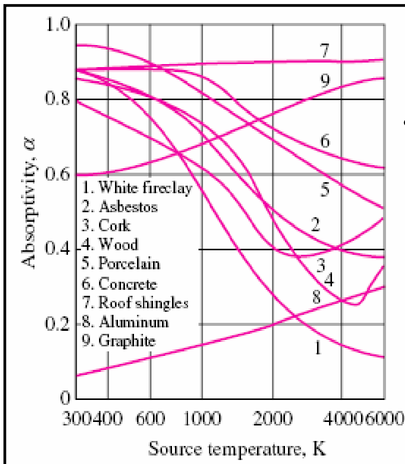


## α Data

- Solar radiation has effective source temperature of about 5800 K

Figure 12-33 from Çengel, *Heat and Mass Transfer*

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## Kirchoff's Law

- Absorptivity equals emissivity (at the same temperature)
- True only for values in a given direction and wavelength
- Assuming total hemispherical values of α and ε are the same simplifies radiation heat transfer calculations, but is not always a good assumption

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## Effect of Temperature

- Emissivity, $\varepsilon$, depends on surface temperature
- Absorptivity, $\alpha$, depends on source temperature (e.g. $T_{sun} \approx 5800$ K)
- For surfaces exposed to solar radiation
  - high $\alpha$ and low $\varepsilon$ will keep surface warm
  - low $\alpha$ and high $\varepsilon$ will keep surface cool
  - Does not violate Kirchoff's law since source and surface temperatures differ

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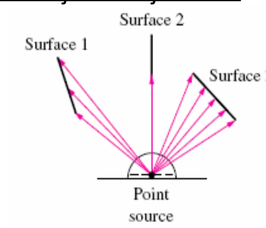
## View Factor, $F_{i\to j}$ or $F_{ij}$

- $F_{i\to j}$ or $F_{ij}$ is the fraction of radiation, leaving surface i, that strikes surface j
  - $A_iF_{ij} = A_jF_{ji}$
  - $\Sigma_k F_{ik} = 1$ (enclosure)
  - $F_{1\to 2+3} = F_{12} + F_{13}$
- $F_{k\to k} = 0$ only if k is a flat surface
- View factors from equations or charts




California State University Northridge Figure 13-1 from Çengel, *Heat and Mass Transfer* 98

## All Black Surface Enclosure

- Heat transfer from surface 1 reaching surface 2 is $A_1F_{12}\sigma T_1^4$
- Heat transfer from surface 2 reaching surface 1 is $A_2F_{21}\sigma T_2^4 = A_1F_{12}\sigma T_2^4$
- Net heat exchange between surface 1 and surface 2: $A_1F_{12}\sigma(T_1^4 - T_2^4)$
  - Negative value indicates heat into surface 1
  - For multiple surfaces

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{ij} \sigma\left(T_i^4 - T_j^4\right)$$

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## Gray Diffuse Opaque Enclosure

- Kirchoff's law applies to the average: $\alpha = \varepsilon$ at all temperatures
- For opaque surfaces $\tau = 0$ so $\alpha + \rho = 1$
- For gray, diffusive, opaque surfaces then $\rho = 1 - \alpha = 1 - \varepsilon$
- Define radiosity, $J = \varepsilon E_b + \rho G$ = emitted and reflected radiation

$$\dot{Q}_i = \frac{A_i\varepsilon_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right) = \frac{E_{bi} - J_i}{R_i} \quad where \quad R_i = \frac{1-\varepsilon_i}{A_i\varepsilon_i}$$

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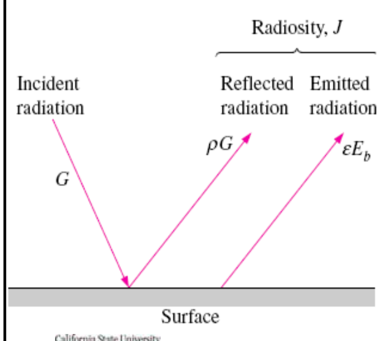
## Net Radiation Leaving Surface




- $\dot{Q} = A(J - G)$
- Can show

$$\dot{Q} = \frac{A\varepsilon}{1-\varepsilon}\left(E_b - J\right)$$

$$\dot{Q}_i = \frac{E_{b,i} - J_i}{R_i}$$

$$R_i = \frac{1-\varepsilon_i}{A\varepsilon_i}$$

California State University Northridge Figure 13-20 from Çengel, *Heat and Mass Transfer* 101

## Gray Diffuse Opaque II

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{ij} = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \qquad R_{ij} = \frac{1}{A_i F_{ij}}$$

- Combining two equations for $\dot{Q}_i$

$$\sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} = \frac{E_{bi} - J_i}{R_i} \quad \Rightarrow \quad \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} + \frac{J_i - E_{bi}}{R_i} = 0$$

- Solve system of N simultaneous linear equations for N values of $J_i$
- Black or reradiating surface ($\dot{Q}_i = 0$) has $J_i = E_{bi} = \sigma T_i^4$

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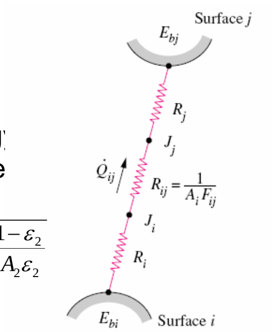
## Review Circuit Analogy

- Look at simple enclosure with only two surfaces
- Apply circuit analogy with total resistance

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_{Total}} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

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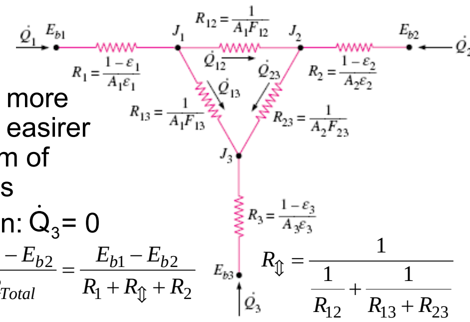
## Three-Surface Circuit




- Three or more surfaces easier by system of equations
- Exception: $\dot{Q}_3 = 0$

$$\dot{Q}_{net,12} = \frac{E_{b1} - E_{b2}}{R_{Total}} = \frac{E_{b1} - E_{b2}}{R_1 + R_\Updownarrow + R_2} \qquad R_\Updownarrow = \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}}$$

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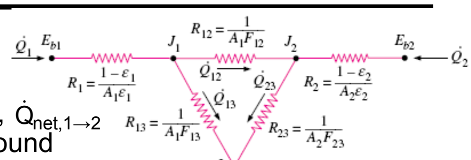
## Review Three-Surface Circuit




- If $\dot{Q}_3 = 0$, $\dot{Q}_{net,1\to 2}$ can be found from circuit with two parallel resistances

$$\dot{Q}_{net,12} = \frac{E_{b1} - E_{b2}}{R_{Total}} = \frac{E_{b1} - E_{b2}}{R_1 + R_\Updownarrow + R_2} \qquad R_\Updownarrow = \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}}$$

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## Radiation Exchange

- Two possible surface conditions: (1) known temperature, (2) known $\dot{Q}_i$

$$\dot{Q}_i = \frac{A_i\varepsilon_i}{1-\varepsilon_i}\left(E_{b_i} - J_i\right) = \sum_{j=1}^{N} A_iF_{ij}\left(J_i - J_j\right) \quad i = 1, \ldots, N$$

(1) $$\left(1 + \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N} F_{ij}\right)J_i - \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N} F_{ij}J_j = E_{b_i} = \sigma T_i^4$$

(2) $$\left(\sum_{j=1, j\neq i}^{N} A_iF_{ij}\right)J_i - \sum_{j=1, j\neq i}^{N} A_iF_{ij}J_j = \dot{Q}_i$$

Solve this set of N simultaneous equations for N values of $J_i$

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## Radiation Exchange II

- Once all $J_i$ values are known we can compute unknown values of $T_i$ and $\dot{Q}_i$
  - For known $T_i$

$$\dot{Q}_i = \frac{A_i\varepsilon_i}{1-\varepsilon_i}\left(E_{b_i} - J_i\right) = \frac{A_i\varepsilon_i}{1-\varepsilon_i}\left(\sigma T_i^4 - J_i\right)$$

  - For known $\dot{Q}_i$

$$E_{b_i} = J_i + \frac{1-\varepsilon_i}{A_i\varepsilon_i}\dot{Q}_i \quad \Rightarrow \quad T_i = \frac{1}{\sigma}\sqrt[4]{J_i + \frac{1-\varepsilon_i}{A_i\varepsilon_i}\dot{Q}_i}$$

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## Numerical Heat Transfer

- Finite difference expressions with truncation error
- Computers give roundoff error
- Convert differential equations to algebraic equations
  - Solve system of algebraic equations to get temperatures at discrete points
  - Reduce step size for stability
- Will not be covered on final

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