8 Snapshots From the Boundaries

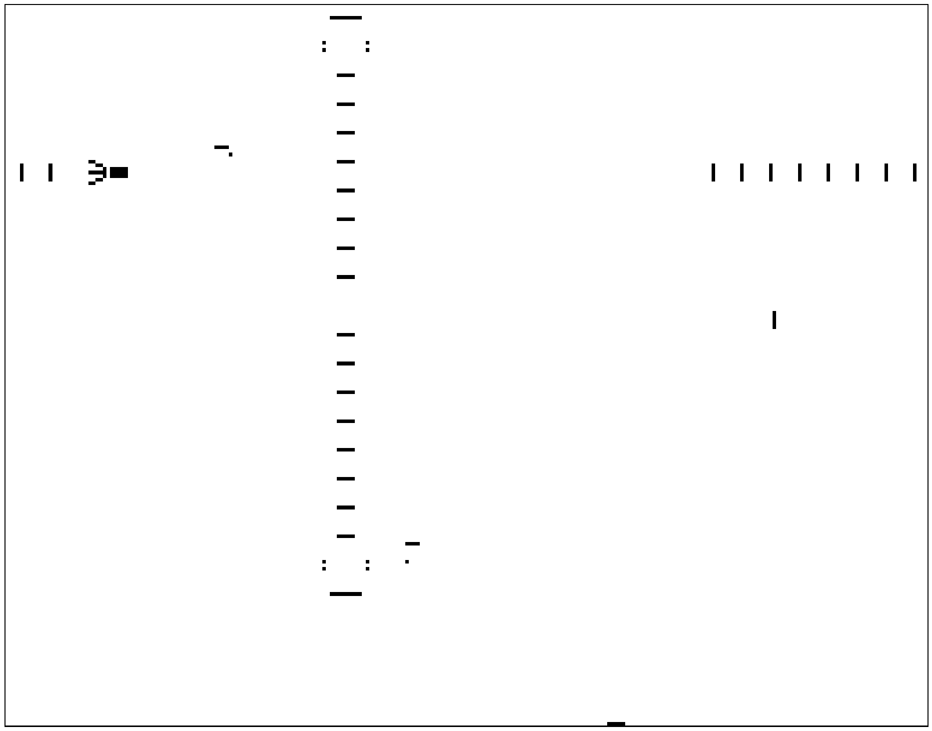
In this chapter we present several snapshots of LtL rules whose ergodic classifications were described quantitatively in Section 3.2. Each of these rules warrant *qualitative* distinctions, however, either because of the dynamics they exhibit on the way to their limiting states, or because their limiting states provide interesting geometries not captured in the quantitative definitions. We seek techniques, perhaps along the lines of those used in [CH], which will enable us to make the following qualitative notions more precise.

8.1 Bugs with trails

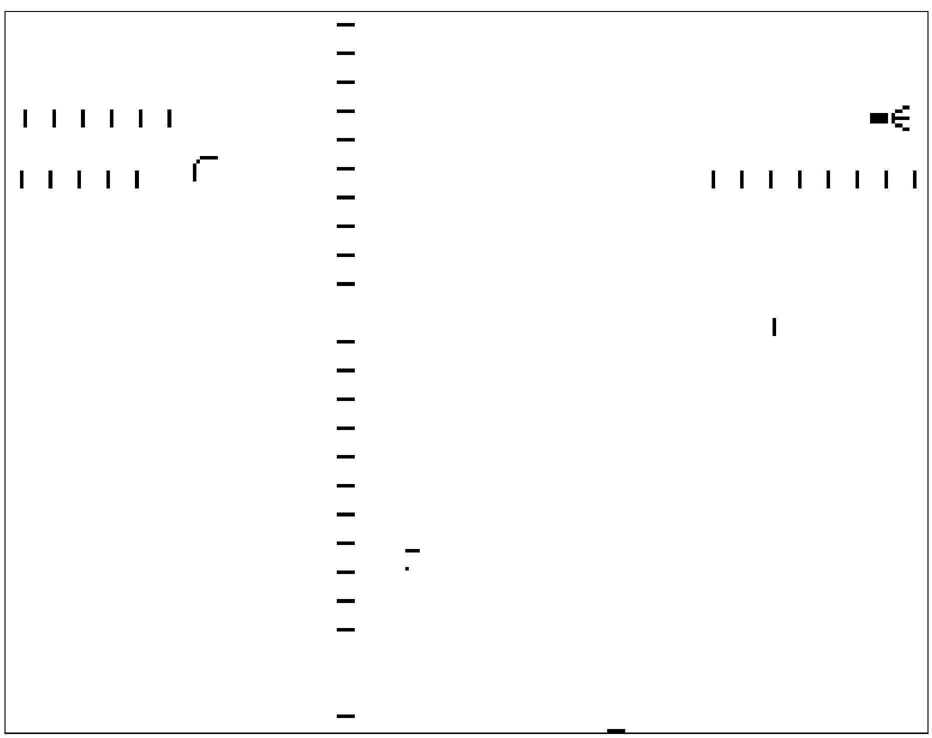
We begin with the range 5 buggin' rule $(5, 9, 9, 5, 5)$. This rule is distinct from other buggin' rules that we have discussed because its bugs generate trails of live sites. The dynamics eventually fixate, however, the bugs' trails provide clues as to how the eventual pattern arose.

The following figures show that by time 50 the only remaining live sites are part of one of the trails generated by the bugs. Each trail appears as a sequence of parallel line segments. Three bugs, which are leaving the trails, are also apparent at time 50. Two are moving in the vertical direction and one in the horizontal direction, each at the end of one of the trails. By time 93 the bugs that had been moving in the vertical direction have collided and annihilated each other. The bug that had been moving in the horizontal direction remains and there is also a bug that is invariant mod translation (like those discussed in section 7.2), in the upper left corner of the figure. Fixation is reached by time 130, with trails indicating the trajectories along which the bugs had previously moved. The portions of the trails that appear at time 93 but not at time 130 were destroyed through a collision with the invariant bug.

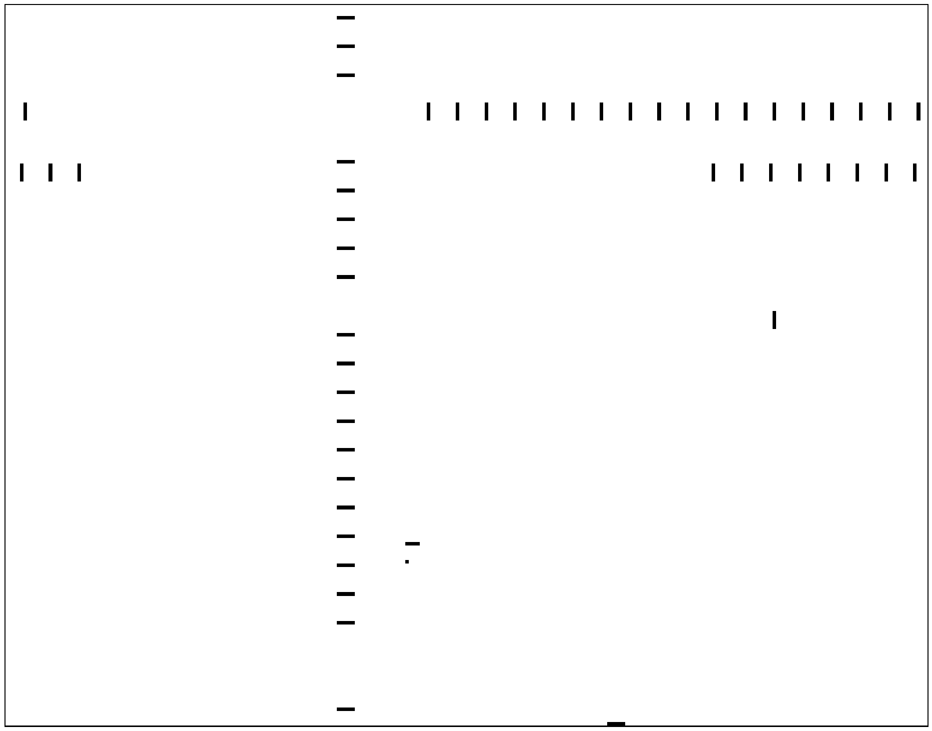
To generate the following figures, we used product measure with density $p = 1/10$ as the initial state, a system size of 256×200, and wrap around boundary conditions.



time 50



time 93

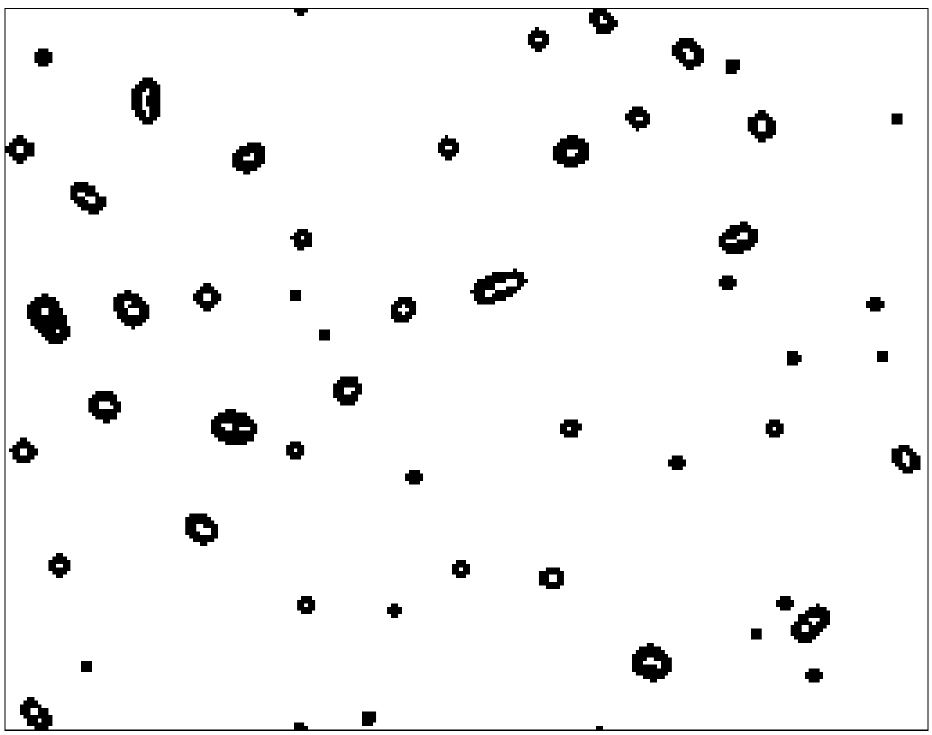


time 130

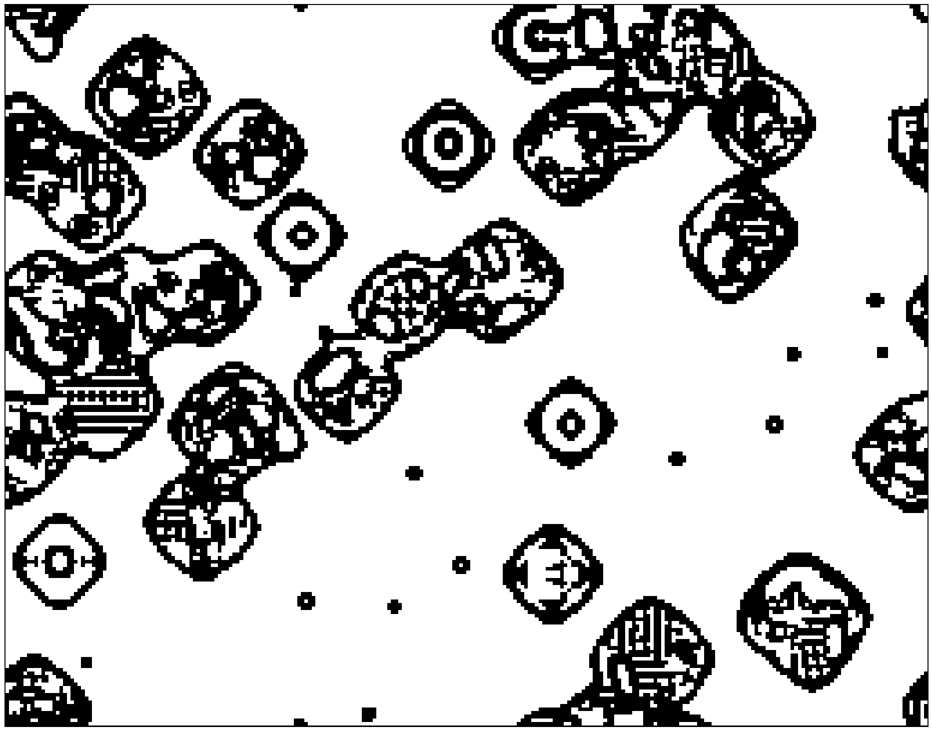
8.2 Slow convergence

We present the next set of snapshots to illustrate an important theme that occurs throughout LtL phase space: the closer a rule is to a phase boundary the longer it takes to reach its limiting state. The example we use is the range 2 rule $(2, 9, 16, 9, 20)$ which lies on a boundary between aperiodic and locally periodic dynamics. Its limiting state is locally periodic but it takes significantly longer to attain that state than nearby rules. For example, if we use product measure with density $p = 1/5$ as the initial state, a system size of 256×200, and wrap around boundary conditions, it takes this rule 2814 time steps to reach its locally periodic limiting state; this is illustrated in the following figures. On the other hand, using the same initial state the rule $(2, 9, 15, 9, 20)$ takes fewer than 150 time steps to reach its locally periodic limiting state and the rule $(2, 9, 16, 9, 21)$ takes fewer than 125 time steps.

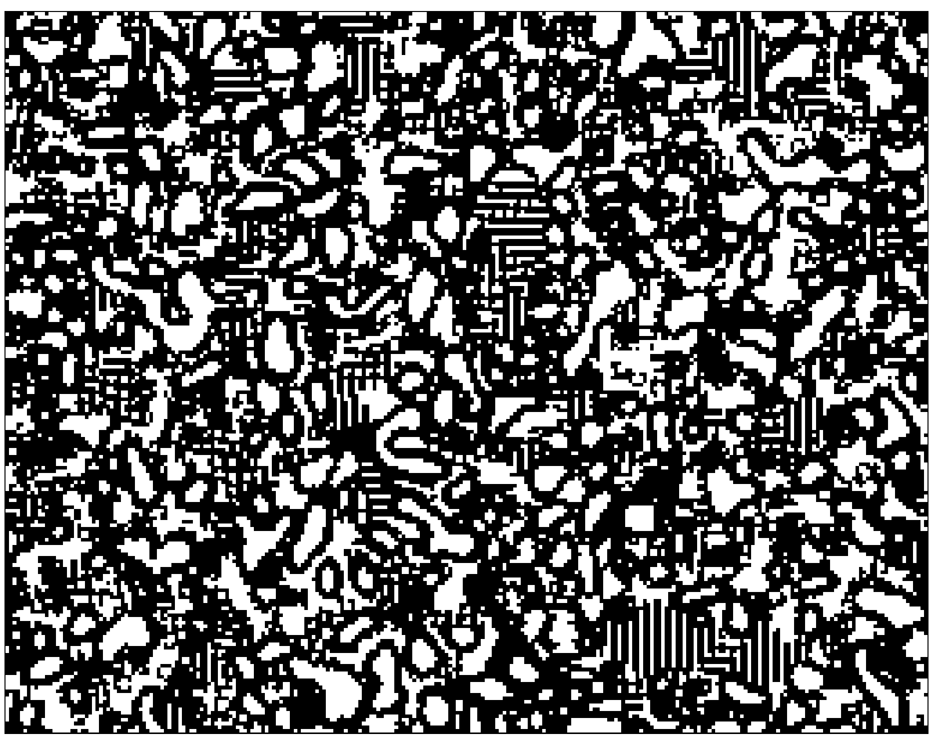
As can be seen in the figures below, by time 5 the only live sites are in finite disjoint configurations. These configurations grow so that by time 25 the density of live sites has inceased. The stripes that appear are regions where the live sites are fixed but not necessarily for all times. This is because the regions where the dynamics are aperiodic continue to grow. By time 300 the regions of live sites occupy the entire lattice. Most of the regions consist of aperiodic dynamics, but some fixed vertical and horizontal alternating stripes of live and dead sites are apparent. At time 600 we see that some of the fixed regions from time 300 have grown while others have been taken over by aperiodic regions. The majority of the fixed regions from time 600 survive, however, and eventually the aperiodic dynamics succumb to the fixed pattern, this is apparent in the figure from time 2814.



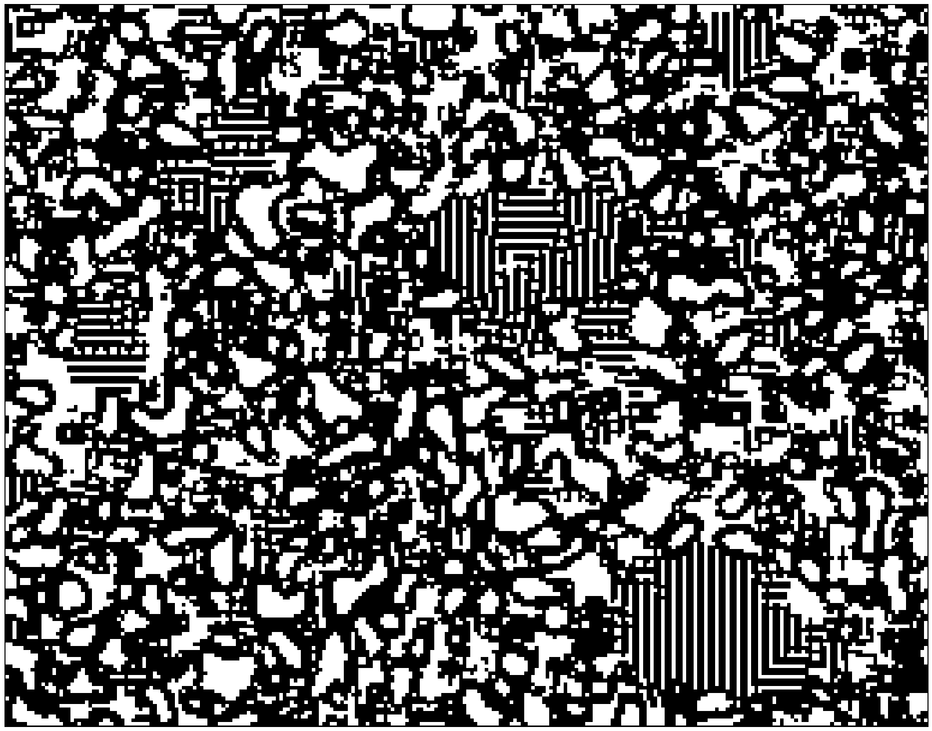
time 5



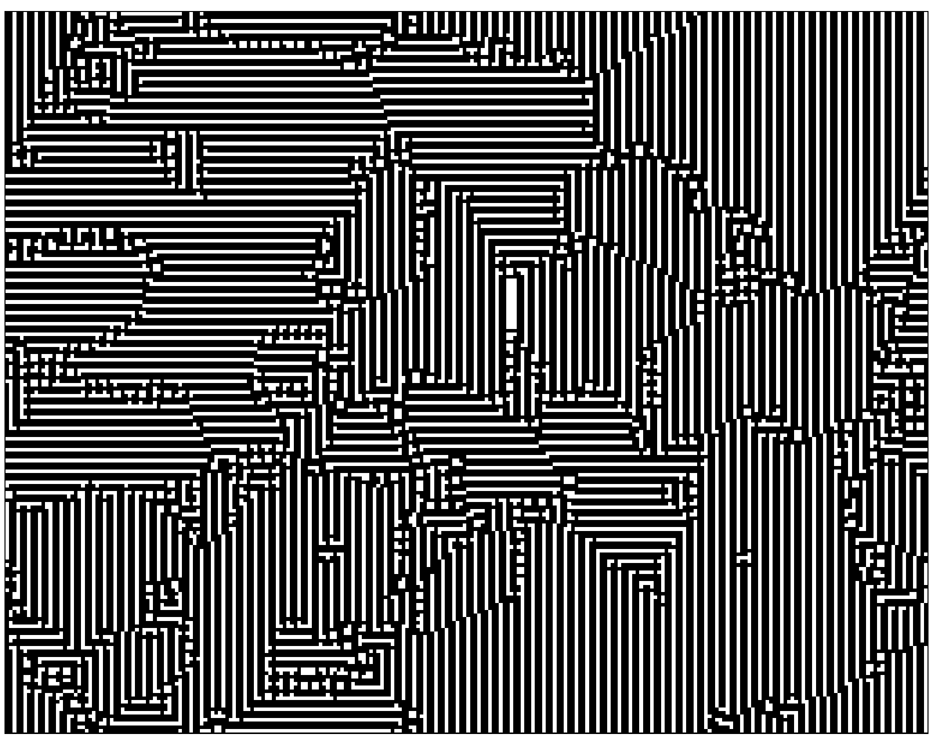
time 25



time 300

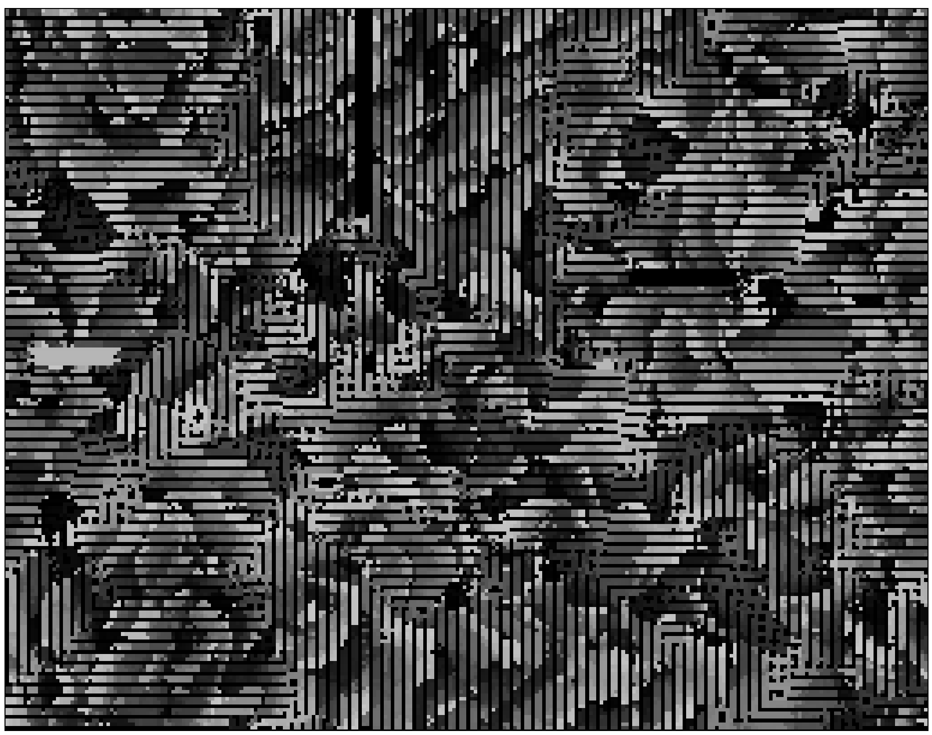


time 600



time 2814

The gometry of the limiting state of the rule (2, 9, 16, 9, 20) is noteworthy as well. As can be seen in the figure from time 2814, it consists of large regions of vertical and horizontal stripes of live and dead sites. As was discussed in section 5.1, an infinite pattern composed of either the vertical or horizontal stripes is an infinite still life for the rule. To illustrate how the eventual pattern of the limiting state evolved, we ran the rule on another initial state of product measure with density 1/5 and kept track of the time at which each site became live. We did this by assigning distinct shades of gray to the sites depending on when they turned on. Using this initial product measure, it took 3852 time steps for the limiting state to be reached. Let us depict that limiting state.

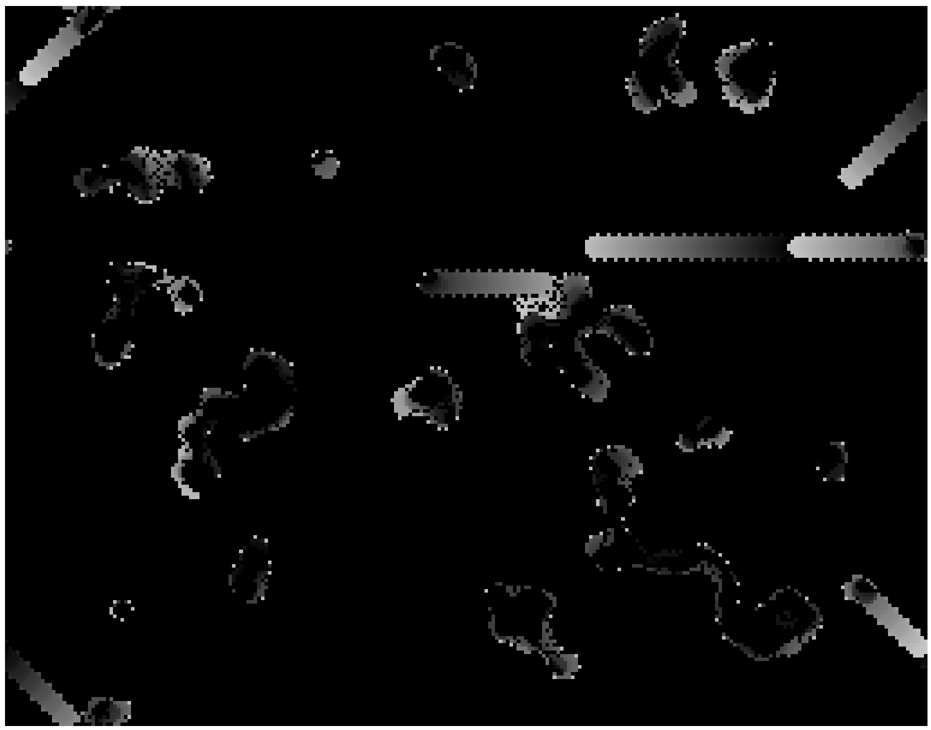


time 3852

8.3 Ladders

The next snapshot was generated by the rule (2, 9, 9, 11, 25) from a random initial state with density 13/23. The *ladders* it generates (depicted below in gray) are viable provided they do not collide with any other configurations of live sites. There is a range one rule known as *Life Without Death* that admits analogous range one ladders, but only those that grow in the horizontal direction. As seen in the following figure this range two rule has ladders that grow in both the horizontal and diagonal directions. The rule's dynamics seem to become nonuniformly locally periodic rather quickly and the ladders appear to be the only sets of $1's$ that grow. On the other hand the Life Without Death rule is more robust, generating intricate snowflake-like patterns which might turn out to grow without bound.

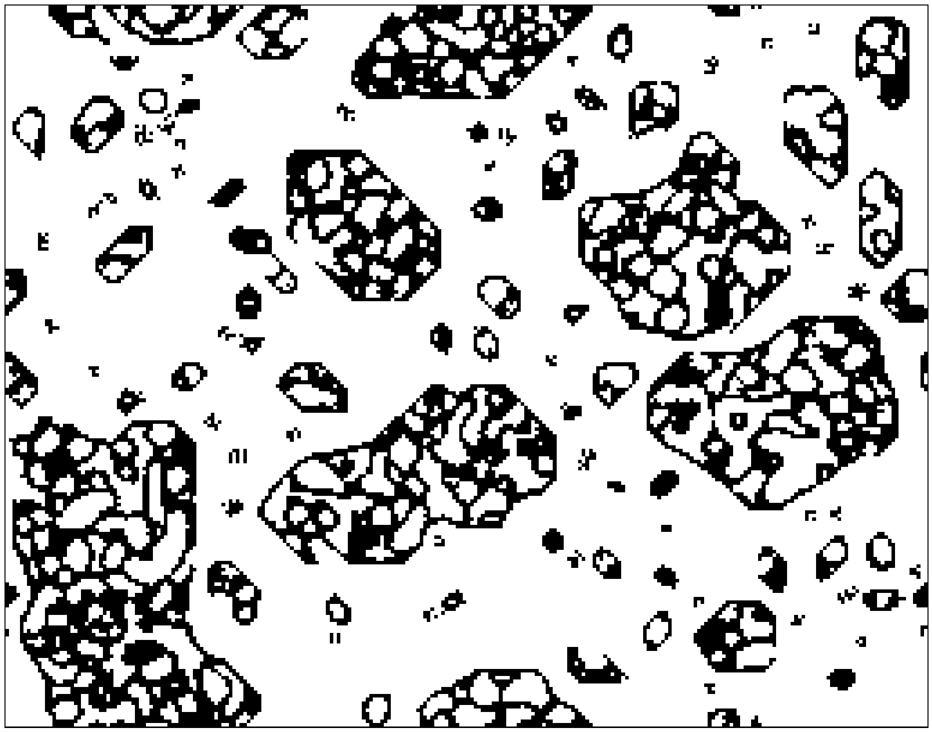
In the following figure live sites are depicted in distinct shades of gray, the shade of each site depending on when it turned on. The result is like a negative of what we have been using $-$ black represents sites that are dead.



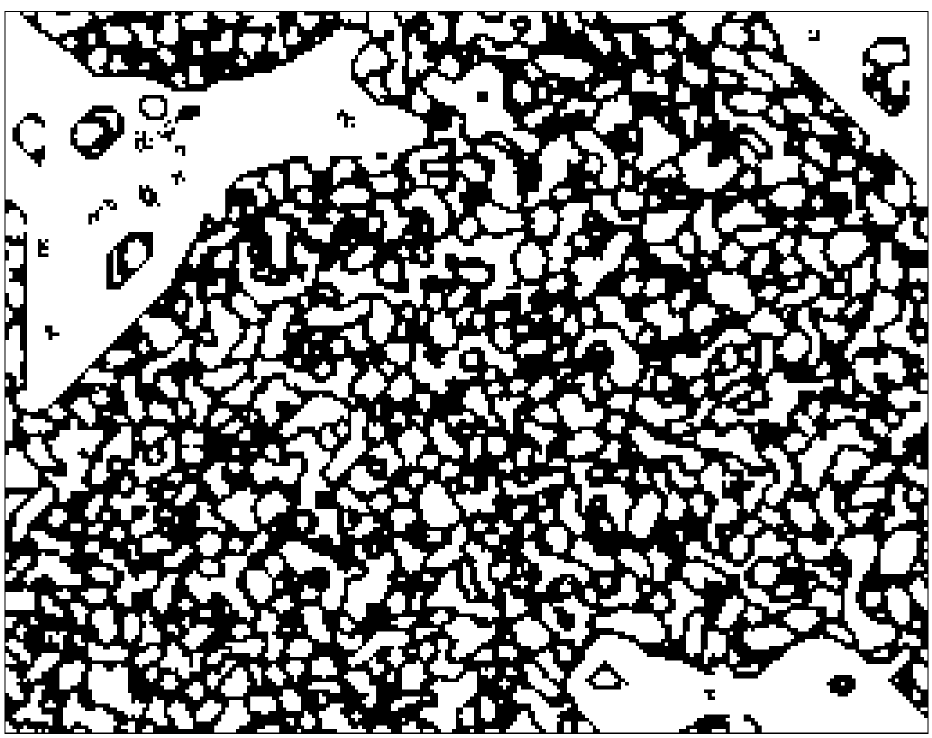
time 389

8.4 Bootstrapping

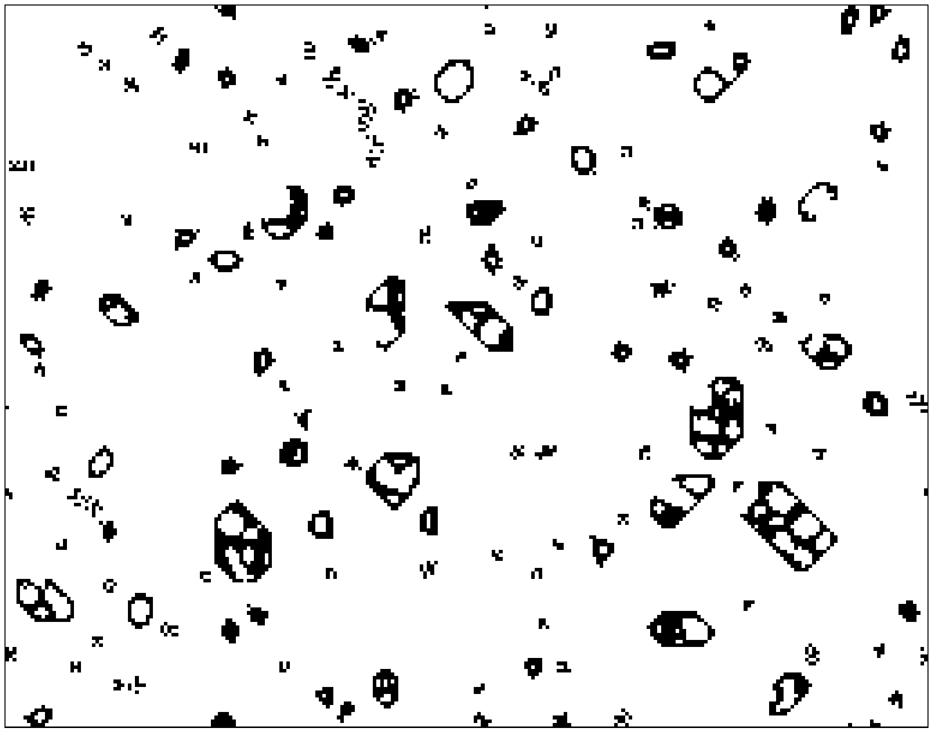
The next snapshot comes from the rule $(2, 11, 18, 6, 18)$ which we conjecture has an aperiodic limiting state. The rule lies on a phase boundary between aperiodic and locally periodic dynamics. If the density of the initial state is large enough, regions of aperiodic dynamics are able to spread by getting help from nearby sets of $1's$, or "bootstrapping." The first two figures illustrate this. Without additional help from the outside, disjoint regions of activity are not able to spread. Rather they are confined to convex sets and eventually become locally periodic. The last two figures in this section show this.



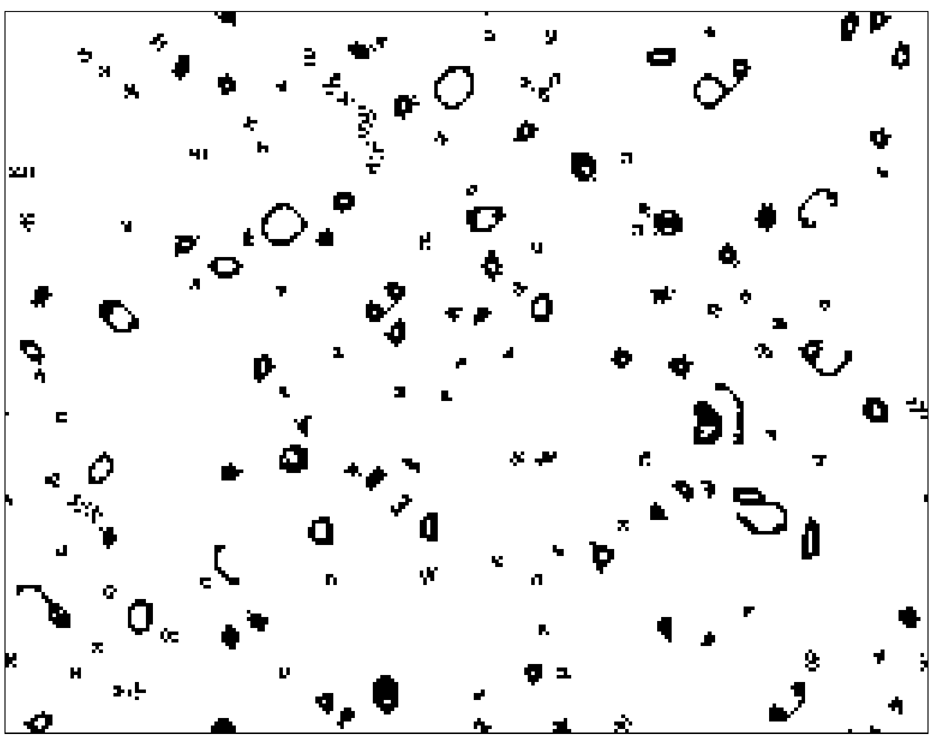
time 100, $p = 5/21$



time 500, $p = 5/21$



time 100, $p = 11/51$

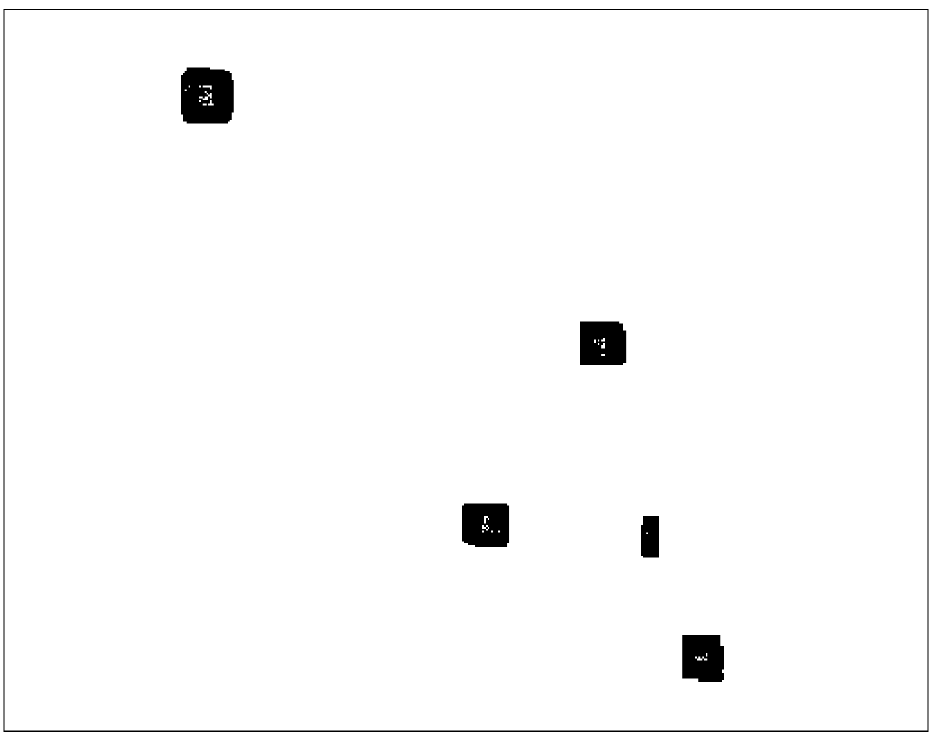


time 3000, $p = 11/51$

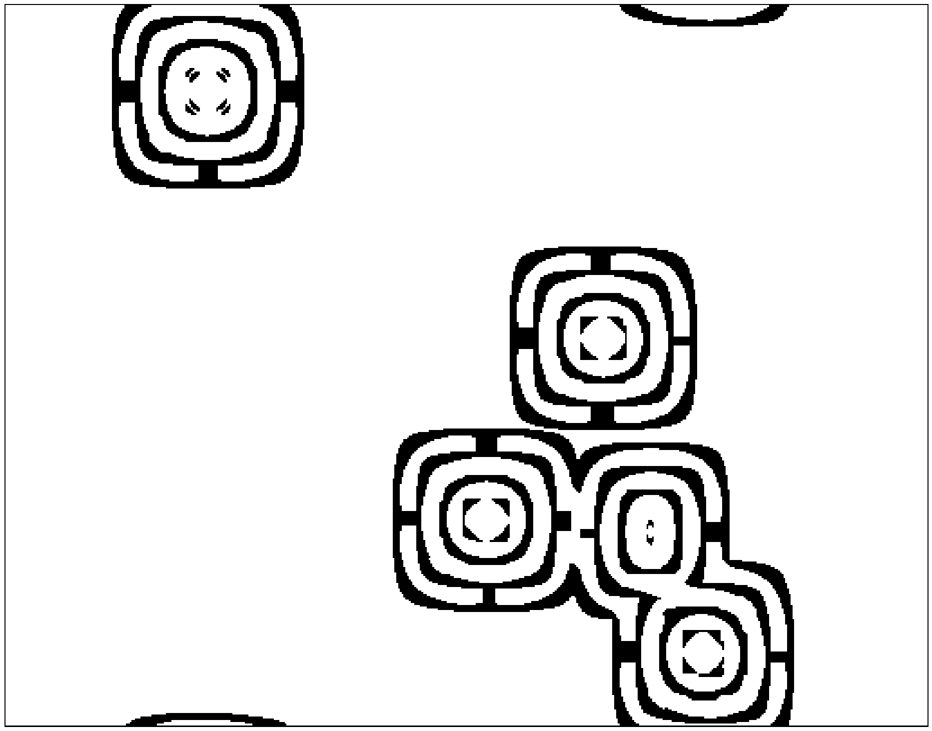
8.5 Self-organized batik

The last snapshot we present comes from the range 13 rule $(13, 10, 100, 100, 320)$. We conjecture that the limiting state of this rule started from product measure with density $1/2$ is nonuniformly locally periodic. Additionally it seems that the period of every site is either 1, 2, or 4. The intrigue is the nucleation as well as resulting geometry that occur when the rule is started from a random initial state.

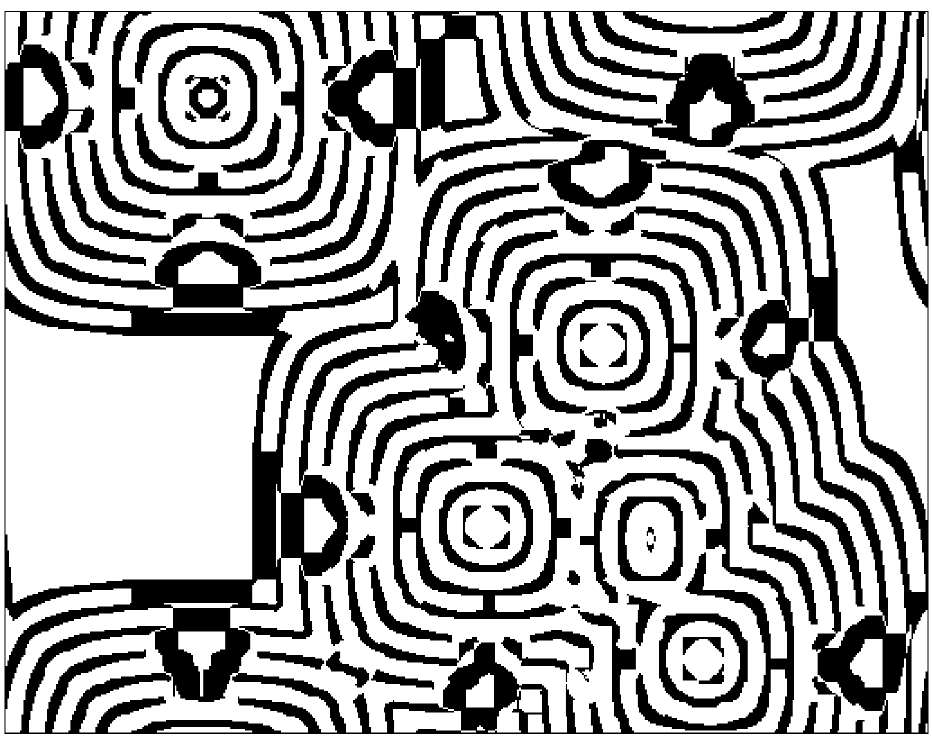
As can be seen in the following figures started from $p = 1/2$ the process quickly relaxes to all $0's$ except for five seeds that form the centers of the nucleating target patterns. The target patterns fill in almost all of the space with concentric waves of alternating 0's and 1's that are fixed. In the gaps between the waves residual pockets of activity continue to fluctuate, some for a very long time, before locking into periodic cycles.



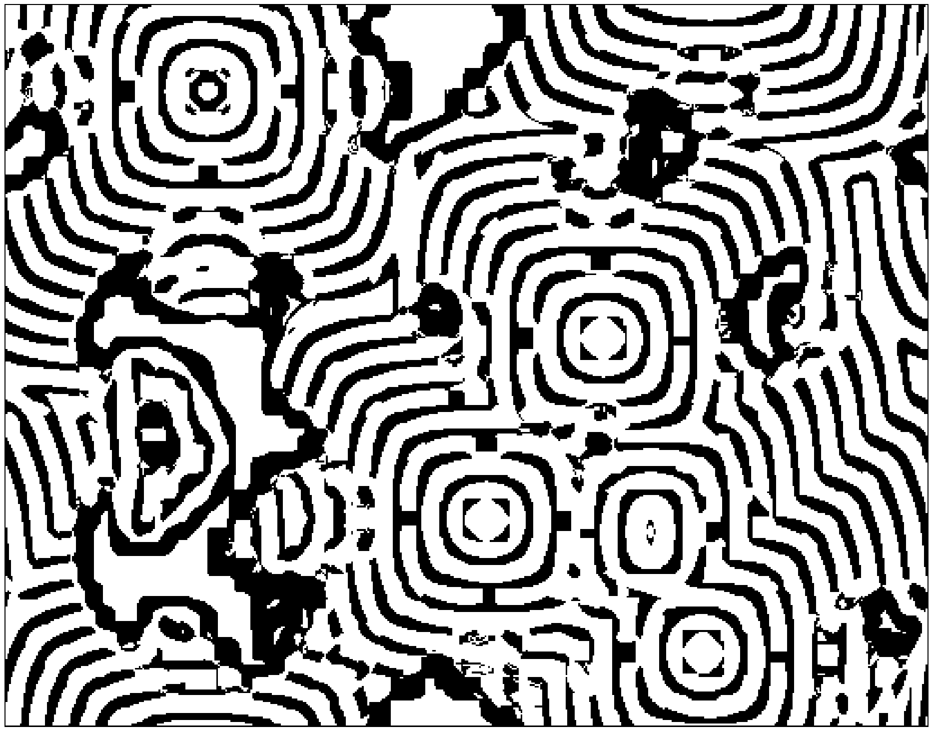
time 2



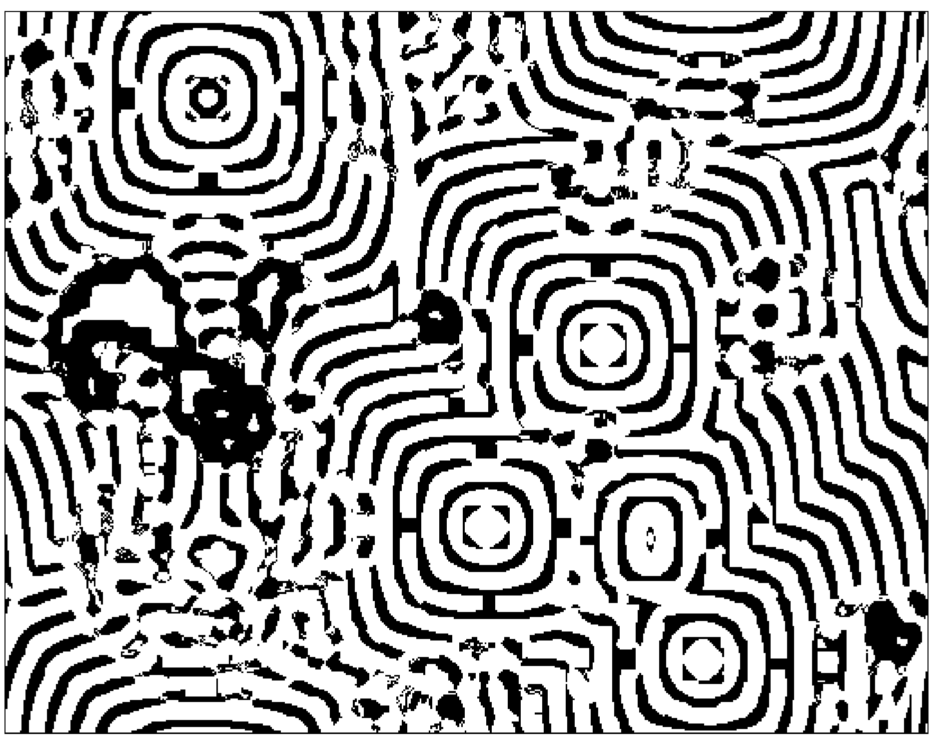
time 5



time 11



time 25



time 75