Appendix II -- Replicators

A different kind of Pascal-generating CA

Example. $d = 2, \mathcal{N} = \{(\pm 2,\ \pm 1), (\pm 1,\ \pm 2)\}$. We include this example to illustrate how the shape of the neighbor set determines the shape of the resulting Pascal-like structure. The space-time diagram which results in this case is quite a bit more complicated than the previous two. We depict a few time steps of the action of f (empty sites have not yet seen any occupied sites in their neighborhoods) and then we show the modular out version.

				⋮				
				0				
		...	0	1	0	...		
				0				
				⋮				

$t = 0$

			1		1			
		1				1		
				0				
		1				1		
			1		1			

$t = 1$

		1		2		1		
	2		2		2		2	
1			0	2	0			1
	2	0	2		2	0	2	
2		2		8		2		2
	2	0	2		2	0	2	
1			0	2	0			1
	2		2		2		2	
		1		2		1		

$t = 2$

			1		3		3		1			
		3		6		6		6		3		
	3		3	0	6	0	6	0	3		3	
1		3	0	9	0	6	0	9	0	3		1
	6	0	9		21	0	21		9	0	6	
3		6	0	21	0	12	0	21	0	6		3
	6	0	6	0	12	0	12	0	6	0	6	
3		6	0	21	0	12	0	21	0	6		3
	6	0	9		21	0	21		9	0	6	
1		3	0	9	0	6	0	9	0	3		1
	3		3	0	6	0	6	0	3		3	
		3		6		6		6		3		
			1		3		3		1			

$t = 3$

				1		4		6		4		1				
			4		12		16		16		12		4			
		6		12	0	18	0	24	0	18	0	12		6		
	4		8	0	24	0	28	0	28	0	24	0	8		4	
1		12	0		0	52	0	54	0	52	0		0	12		1
	12	0	24	0	72	0	84	0	84	0	72	0	24	0	12	
4		18	0	52	0	54	0	96	0	54	0	52	0	18		4
	16	0	28	0	84	0	96	0	96	0	84	0	28	0	16	
6		24	0	54	0	96	0	168	0	96	0	54	0	24		6
	16	0	28	0	84	0	96	0	96	0	84	0	28	0	16	
4		18	0	52	0	54	0	96	0	54	0	52	0	18		4
	12	0	24	0	72	0	84	0	84	0	72	0	24	0	12	
1		12	0		0	52	0	54	0	52	0		0	12		1
	4		8	0	24	0	28	0	28	0	24	0	8		4	
		6		12	0	18	0	24	0	18	0	12		6		
			4		12		16		16		12		4			
				1		4		6		4		1				

$t = 4$

Ignoring the $0's$ and modding out by 2 gives:

```
1
```

$t = 0$

```
  1 1
1     1

1     1
  1 1
```

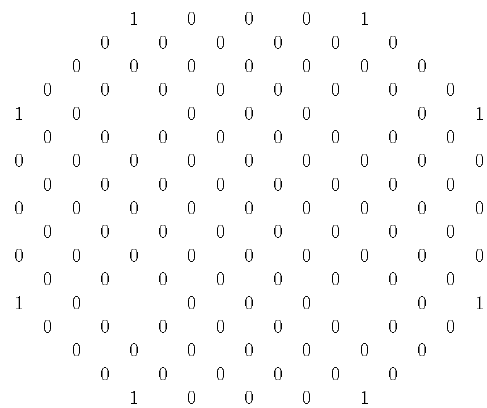
$t = 1$

```
    1   0   1
  0   0   0   0
1       0       1
  0   0   0   0
0   0   0   0   0
  0   0   0   0
1       0       1
  0   0   0   0
    1   0   1
```

$t = 2$

```
      1   1   1   1
    1   0   0   0   1
  1   1   0   0   1   1
1   1   1   0   1   1   1
  0   1   1   1   1   0
1   0   1   0   1   0   1
  0   0   0   0   0   0
1   0   1   0   1   0   1
  0   1   1   1   1   0
1   1   1   0   1   1   1
  1   1   0   0   1   1
    1   0   0   0   1
      1   1   1   1
```

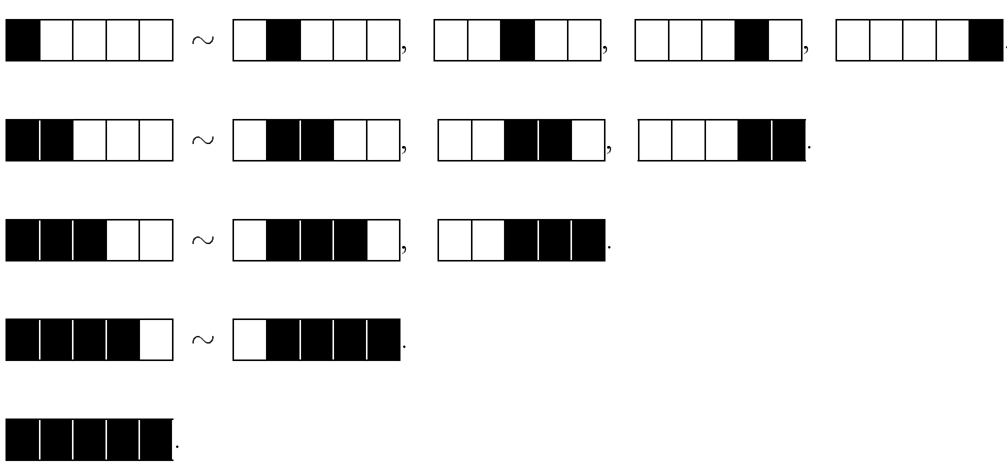
$t = 3$

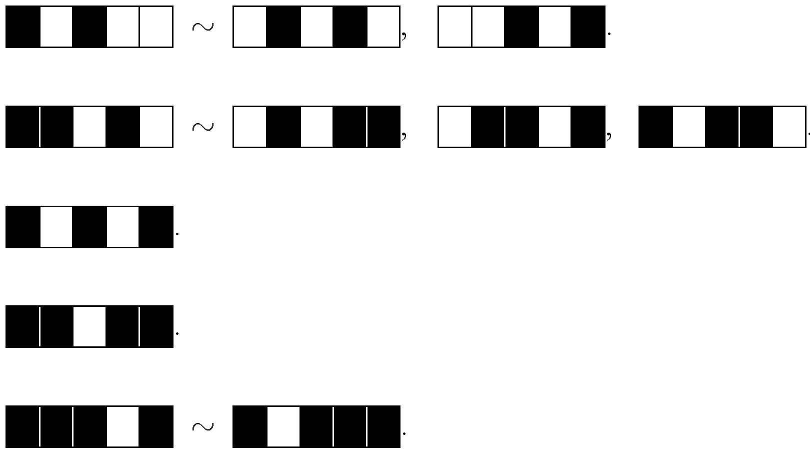


$t = 4$

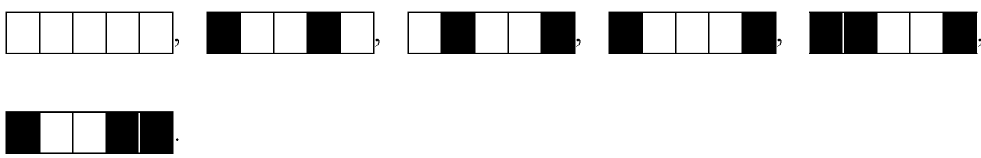
A counting problem

This section shows why there are only 10 relevant configurations to check for the string of length 5 in Example 6.3.2. For each of the 10 relevant strings, we illustrate the equivalence class (with respect to translation or 180° rotation) that it represents. Following the set of equivalence classes, we illustrate the set of irrelevant strings (which are thus because they evolve as unions of the relevant strings).





Irrelevant strings:

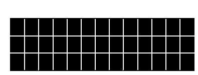


Replicator Collection

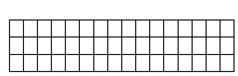
The following examples satisfy the axioms listed in Section 6.2. In each case, the $k_i's$ from Theorem 6.2.1 are computed.

One-dimensional replicators

Example 1. LtL rule $(7, 14, 15, 14, 15)$. In this case, Λ is the 13×3 rectangle,



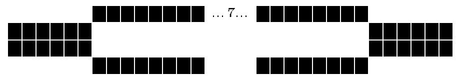
$\mathcal{I}$ is the 16 $\times$ 3 rectangle,



$\tau = 2$, and $\nu = 1$. Thus, $\lambda_1 = 13$ and $\sigma_1 = 16$. Since $\rho = 5$, Theorem 6.2.1 gives

$$k_1 = \left\lfloor \frac{2(5)(2)-(16-13)}{16} \right\rfloor = 1.$$

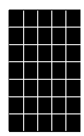
Example 2. LtL rule $(7, 14, 14, 14, 14)$. In this case, Λ is the configuration,



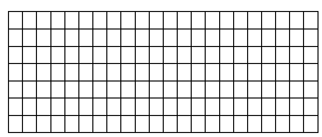
$\mathcal{I}$ is a 154 $\times$ 4 rectangle, $\tau = 22$, and $\nu = 1$. Thus, $\lambda_1 = 35$ and $\sigma_1 = 154$. Since $\rho = 7$, Theorem 6.2.1 gives

$$k_1 = \left\lfloor \frac{2(7)(22) - (154 - 35)}{154} \right\rfloor = 1.$$

Example 3. LtL rule $(5, 27, 32, 26, 44)$. In this case, Λ is the 5×7 rectangle,



$\mathcal{I}$ is the 22 $\times$ 7 rectangle,



$\tau = 11$, and $\nu = 1$. Thus, $\lambda_1 = 5$ and $\sigma_1 = 22$. Since $\rho = 5$, Theorem 6.2.1 gives

$$k_1 = \left\lfloor \frac{2(5)(11) - (22 - 5)}{22} \right\rfloor = 4.$$

Two-dimensional replicators

Example 4. The *exactly* θ LtL rule $(\rho, \theta, \theta, \theta, \theta)$, for $\theta = \rho - k,\ k \in \{0, 1, 2, ..., \rho - 1\}$. In this case, Λ is a $1 \times (2\rho + 1)$ rectangle, $\mathcal{I}$ is a square with side of length $2(\rho + k + 1)$, $\tau = 2$, and $\nu = 2$. Thus, $\lambda_1 = 1,\ \lambda_2 = 2\rho + 1,$ and $\sigma_1 = \sigma_2 = 2(\rho + k + 1)$. Theorem 6.2.1 gives

$$k_1 = \left\lfloor \frac{2(\rho)(2) - (2(\rho + k + 1) - 1)}{2(\rho + k + 1)} \right\rfloor = \left\lfloor \frac{2\rho - 2k - 1}{2(\rho + k + 1)} \right\rfloor = 0 \text{ and}$$

$$k_2 = \left\lfloor \frac{2(\rho)(2)-(2(\rho+k+1)-(2\rho+1))}{2(\rho+k+1)} \right\rfloor = \left\lfloor \frac{4\rho-2k-1}{2(\rho+k+1)} \right\rfloor = \left\lfloor \frac{(2\rho+2k+2)+2\rho-4k-3}{2(\rho+k+1)} \right\rfloor = 1.$$

Times $2t$, $t = 0, 1, ..., 8$ for the $\rho = \theta = 5$ case, are depicted in Section 4.4. In that example, $\lambda_1 = 1$, $\lambda_2 = 11$, and $\sigma_1 = \sigma_2 = 12$. The above also holds for the rules $(\rho, \theta, \theta, l, l)$, for $\theta = \rho - k$, $k \in \{0, 1, 2, ..., \rho - 1\}$, and $0 \le l \le \theta$.

Example 5. LtL rule $(\rho, \theta, \theta, \theta, \theta)$, for $\theta = \rho + 1$. In this case, Λ is a $1 \times (2\rho + 1)$ rectangle, $\mathcal{I}$ is a square with side of length 10ρ, $\tau = 10$, and $\nu = 2$. Thus, $\lambda_1 = 1$, $\lambda_2 = 2\rho + 1$, and $\sigma_1 = \sigma_2 = 10\rho$. Theorem 6.2.1 gives

$$k_1 = \left\lfloor \frac{2(\rho)(10)-(10\rho-1)}{10\rho} \right\rfloor = 0 \text{ and}$$

$$k_2 = \left\lfloor \frac{2(\rho)(10)-(10\rho-(2\rho+1))}{10\rho} \right\rfloor = \left\lfloor \frac{12\rho+1}{10\rho} \right\rfloor = 1.$$