

If quantity "a" of item x and "b" of item y are related (so there is some reason to compare the numbers), we say the RATIO of x to y is "a to b", written "a:b" or  $a/b$ .

Ex 1 If a classroom of 21 *students* has 3 *computers* for student use, we say the ratio of *students to computers* is \_\_\_\_\_.

We also say the ratio of *computers to students* is 3:21.

The ratio, being defined as a fraction, may be reduced. In this case, we can also say there is a 1:7 ratio of computers to students (or a 7:1 ratio of students to computers). We might also say "there is one computer per seven students".

Ex 2 We express gasoline consumption by a car as a ratio: If we traveled 372 miles since the last fill-up, and needed 12 gallons to fill the tank, we'd say we're getting \_\_\_\_\_ mpg (*miles per gallon—mi/gal*).

We also express speed as a ratio—distance traveled to time elapsed...

Caution: Saying "the ratio of x to y is a:b" *does not mean* that x constitutes  $a/b$  of the *whole*; in fact x constitutes  $a/(a+b)$  of the whole of x and y together. For instance if the ratio of boys to girls in a certain class is 2:3, *boys* do not comprise  $2/3$  of the *class*, but rather \_\_\_\_\_!

**A PROPORTION is an equality of two ratios.**

Ex 3 If our car travels 500 miles on 15 gallons of gas, how many gallons are needed to travel 1200 miles? We believe the fuel consumption we have experienced is the most likely predictor of fuel use on the trip. We assume that the mpg ratio of our trip will equal mpg of the past. Therefore we write the *proportion*:

$$500 \text{ miles}:15 \text{ gallons} = 1200 \text{ miles}:x \text{ gallons} \quad (\text{or } 500:15 = 1200:x).$$

Or, expressed in the more familiar form:  $500/15 = 1200/x$  ...or  $x/1200 = 15/500$  ...).

Ex 4 If notation on the corner of a map indicates a scale of  $1/2$  inch per 12 miles. A distance of 5 and  $1/2$  inches on the map corresponds to \_\_\_\_\_ miles.

Ex 5 Tea brand A sells in a 6-ounce container for \$3; Brand B sells in a 10-ounce container for \$4. Are these equivalent price-per-ounce?

Ex 6 There are 105 male babies born for every 100 girl babies. Out of 20,000 babies born in LA last month, approximately how many were girls? \_\_\_\_\_

Ex 7 One-fourth of a company's employees were laid off. If 552 remained, how many were laid off? What was the company's original work-force?

Ex 8 In 1970, 4 men to every 1 woman were admitted to med-school. Of 850 admissions, how many were male?

Ex 9 A recipe for champagne punch calls for 5 parts ginger ale, 5 parts champagne and 1 part limeade. How much champagne is needed to make 500 4-ounce servings?

Ex 10 The "standard toll-house" cookie recipe calls for 1 cup chocolate chips & 1 c nuts to 4 c flour. Julia makes hers with 2 cups chocolate chips and 1 cup nuts to four cups flour. Jacques composes his of 1 cup chocolate chips and 1 cup nuts to three cups flour. Which recipe is more true to the proportions established in the "standard" recipe? Whose would you rather eat?

Ans: E1 21:3 or 7:1 E2 31 E3 x is 36 E4 132 E5 B is cheaper E6 9756 21:20 E7 1656 2208 E8 680 E9 ~909 oz E10 compare the fractions. Jacques. Julia's!

Example 1.6 from the text, p 169:

The ratio of boys to girls in a class is 5:3.  
There are 6 more boys than girls.  
How many are in the class?

Example 1.7:

The ratio of Jim's money to Peter's money was 4:7 at first.  
After Jim spent  $\frac{1}{2}$  of his money and Peter spent \$60,  
Peter had twice as much money as Jim.

How much money did Jim have at first?

Algebra solution:

Let  $x$  = Jim's original \$

Then Peter had  $7x/4$

After: Jim then had  $x/2$

Peter then had  $7x/4 - 60$

and that was 2(what Jim had then)  $\frac{7x}{4} - 60 = 2 \left( \frac{x}{2} \right)$

Alternate algebra solution:

Let  $x$  be a certain tidy sum of money (\$).

Since the ratio was 4:7... Jim had  $4x$  and Peter had  $7x$  at first.

After Jim spent  $\frac{1}{2}$  his money, he then had  $2x$ .

After Peter spent \$60, he then had  $7x - 60$ .

We are told at this point Peter had twice as much as Jim, so  $7x - 60 = 2(2x)$

(Compare that to the equation above.)

*Very similar, but not identical because the  $x$  variable above is Jim's money. This  $x$  is  $1/4$  of that. )*

Example 1.11:

Two numbers are in the ratio 3:5.  
After subtracting 11 from each,  
the new ratio is 2:7.  
What are the numbers?

Example 1.14:

A bag contained 6 White & 10 Red marbles.  
4 White & 20 Red marbles were added to the bag.  
What is the new ratio of White to Red marbles?

D, K & M solved this problem differently:

D said  $3:5 + 1:5 = 4:10$

K said  $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

M said  $6+4 : 10+20 = 10:30 = 1:3$

**Ratios do NOT add. We can set ratios = to make a proportion. Ratios DON'T tell us how many total!**

Every rational can be expressed as a ratio (fraction), a decimal or a percent.

*PERCENT* means "per hundred".

Thus **50 percent (50%)** means **50 per hundred**, or **50/100** or **fifty hundredths**, or **.50**.

You must be able to express any one of these forms in any of the other.

★ Express the following PERCENTAGES as rational-form FRACTIONS *and* as DECIMALS:

$$12\% = \frac{\quad}{\quad} = \underline{\quad} \quad 20\% = \frac{\quad}{\quad} = \underline{\quad} \quad 7\% = \frac{\quad}{\quad} = \underline{\quad} \quad 100\% = \underline{\quad}$$

$$250\% = \frac{\quad}{\quad} = \underline{\quad} \quad 1500\% = \frac{\quad}{\quad} = \underline{\quad} \quad 7\frac{1}{2}\% = \frac{\quad}{\quad} = \underline{\quad} \quad \frac{1}{2}\% = \frac{\quad}{\quad} = \underline{\quad}$$

**To change "x percent" to decimal form:**

☆ Since % means "per hundred",  $X\%$  means  $X/100$ .... (we do what "%" says... divide by 100).

★ Express these DECIMALS as PERCENTAGES:

$$.25 = \underline{\quad}\% \quad .04 = \underline{\quad}\% \quad .0425 = \underline{\quad}\% \quad .4 = \underline{\quad}\%$$

$$.002 = \underline{\quad}\% \quad 1.5 = \underline{\quad}\% \quad 320 = \underline{\quad}\% \quad .0003 = \underline{\quad}\%$$

**To change a decimal to percent form:**

☆ % means "per hundred" or "hundredths" so if you can write as 100ths, conversion is intuitive.

☆ Use "patterning": if  $.07 = 7\%$ , then  $.7$  must be  $70\%$ , and  $.007$  must be  $.7\%$ .

☆ Since dividing by 100 converts %  $\rightarrow$  decimal, so multiplying by 100 should convert decimal  $\rightarrow$  %.

★ Express these FRACTIONS as PERCENTAGES:

$$1/2 = \underline{\quad}\% \quad 3/5 = \underline{\quad}\% \quad 1/3 = \underline{\quad}\% \quad 1/8 = \underline{\quad}\% \quad 4/3 = \underline{\quad}\%$$

**To change a fraction to "percent":**

☆ % means hundredths, so write as hundredths:  $3/8 = .375 = 37.5/100 = 37.5\%$

☆ Divide to convert the fraction to decimal form, then convert to % as above.

**% word problems:**

- E1 Susan bought a house in 1994 for 140,000. The value of the house has increased 250%. How much has the value increased? What is the value of the house now?
- E2 John was making 25,000 per year, until he got a raise to 26,000 per year. By what percent was his salary raised?
- E3 Jackie bought a dress on sale at 30% off for \$150. What was the original price?

Thirty percent of 500 is \_\_\_\_\_.

Equation:

One hundred twenty is what percent of eighty? \_\_\_\_\_

Equation:

Thirty is twenty percent of \_\_\_\_\_.

Equation:

0. Express  $10^{-5}$  as a percent.
1. Generic problems:  
 a. Thirty percent of 500 is \_\_\_\_\_. Equation:  
 b. One hundred twenty is what percent of eighty? \_\_\_\_\_ Equation:  
 c. Thirty is twenty percent of \_\_\_\_\_. Equation:
2. *Quick!* 9.25% is: a. .925 b. .0925 c. 9.25  
 What is greater: .10, or 3% of 2.98?
3. If the sales tax rate is 8.25%, and the sales tax on my new car is \$559.35, what is the selling price of the car?
4. In another state, the sales tax on a \$350 item is \$14.00. What is that state's sales tax rate?
5. If the sales tax rate is 7%, how much money is needed to purchase an item whose price is \$3.71?
6. Thirteen of thirty-five students in a calculus class earned A's. What percent of the class earned A's?
7. If the property tax rate is 1.2%, & the tax assessment on a home is \$2,398, what is its assessed value?
8. Thirty percent of immigrants to LA become entrepreneurs; of those, 45% are successful. What percentage of immigrants to LA become successful entrepreneurs?
9. Zeta is currently making \$34,500 per year; she needs to earn at least \$40,000 per year to buy a townhome. What percentage raise does she need?
10. A dress originally priced at \$550.00 is already marked down 30%, and the *red-tag sale sign* says "Additional 50% off marked price". What is the total percent discount?
11. A school lunch program has been selling food without sales tax. The St Board of Equalization demands payment of tax. The school collected \$8,432,200 in all for hot lunches; the tax rate is 7%. How much must the school *fork over*?
12. Marco received a ten percent raise in January, but now that the company is "downsizing", all remaining employees are receiving a ten percent pay cut. What is the effect on Marco's salary, relative to last year's? (Overall, is Marco now making more than, less than, or the same as, last year's pay?) Explain.

Answers:

0. .001% 1. 150;150;150 2. .0925 ; .10 > .03x3 > ...  
 3. \$6780 4. 4% 5. \$3.97 6. ≈37% 7. \$199833 8. 13.5% 9. 15.942% 10. 65%  
 11. \$551639.25 (It's not 7% of 8432200.)

(Here's an example that's closer to Home! Suppose you are at a Dodger game, and you buy a hot dog from the vendor for \$3. Was that transaction subject to sales tax? How does that work; do you think the vendor is going to send 7% of \$3 to the state?)

12. Marco will end up with a 1% decrease in pay compared to last year.

(That's because he received a ten percent pay raise, making his new salary 110% of last year's... then lost 10% of 110% = **11% of last year's pay**. Marco would be better off if he had said forget about giving me a raise!)