V1

1	2	3	4	5	6

Last Name:________________

First Name:________________

ID:________________ Section:____

Math 150a Midterm #3. April 29, 2005

Attention! Please, note that this is the closed book test. You are not allowed to use graphing calculator. Simple calculators are allowed. Please, show all important steps in you solution but do not make your solution excessively long.

1. (15pt) Find the two numbers, x and y, such that $xy = 36$, and the value of $x^2 + y^2$ is a minimum.

2. Sketch the graph of the function $f(x)$ following the steps listed below:

$$f(x) = \ln(x^2 + 1).$$

a)(3pt) specify the domain;

b)(5pt) specify the intervals of increasing and decreasing, list critical points;

c)(5pt) specify the intervals where the function is concave up and concave down, list inflection points;

d)(7pt) check for vertical, horizontal, and oblique asymptotes. Sketch the graph.

3. Find the most general antiderivative for the functions

a) (5pt)

$$f(x) = \frac{1}{\sqrt{1-x^2}};$$

b) (5pt)

$$f(x) = \frac{x^2+1}{x};$$

c) (5pt)

$$f(x) = \frac{1}{1-\sin^2 x}.$$

4. Find the following derivatives:

a) (6pt)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^x \sqrt{1+t^3}\,\mathrm{d}t\right]$$

b) (6pt)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^{\sqrt{x}} \sqrt{1+t^3}\,\mathrm{d}t\right]$$

c) (8pt)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^{\sqrt{x}} x\sqrt{1+t^3}\,\mathrm{d}t\right]$$

5. a) (5pt) Write the following sum in the expanded form. DO NOT CALCULATE!

$$\sum_{i=2}^{6} \frac{1}{n} \sin\left(\frac{i}{n}\right)$$

b) (5pt) Perform the indicated change of variable in the index. DO NOT CALCULATE.

$$\sum_{i=4}^{100} 2^i \sqrt{1+3i}, \qquad k = i - 3$$

c) (5pt) Use the given values of a and b and express the following limit as a definite integral. DO NOT CALCULATE.

$$\lim_{|p|\to 0} \sum_{i=1}^{n} \sqrt{1+(x_i^*)^2}\Delta x_i, \quad a = -1, \quad b = 1.$$

6. Solve the differential equation by separation of variables: find the general solution, then use the given condition to find the value of the constant.
b) (7pt)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^x}{y}, \qquad y = 1 \text{ at } x = 0;$$

b) (8pt)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos^2(y)}{1 + x^2}, \qquad y = 0 \text{ at } x = 0;$$