Functions, Part 1

Section 2

- Introductory Example
- Definition and Notation
- Graphing a Function
- Linear Cost Model

Introductory Example: Fill the gas tank

Your gas tank holds 12 gallons, but right now you're running on empty. As you pull into the gas station, the engine sputters and dies—the gas tank is completely empty. You pump 12 gallons into the tank and swipe your credit card. How much does it cost?

That depends on the price of gas, of course! But exactly how does it depend on the price?

If the price per gallon is $3.40, what is the cost to fill the tank?

If the price per gallon is $4.10, what is the cost to fill the tank?

Fill the gas tank: function and symbolic form

The symbolic form:
Write an algebraic expression for the cost in terms of the price. In our example:

$$C(p) = 12p.$$

This is read as:
The cost, $C(p)$, to fill the tank is a **function** of the price p per gallon.

Written form:
"the cost of filling the tank at a price of \$3.40 per gallon is \$40.80."
Symbolic form:

$$C(\underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$$

Practice translating statements between symbolic form and function notation.

Here are some other examples from the gas tank situation:

Words: At a price of \$2.00 per gallon, it costs \$24.00 to fill the tank.
Write the symbolic form:

Words: The cost to fill a 12-gallon tank at a price of p dollars per gallon is 12 times p.
Write the symbolic form:

Words: What is price per gallon if it costs \$45.00 to fill the tank?
Symbolic form: What is p if $C(p) = 45.00$?
Solve $12p = 45.00$ for p. (We'll do this later.)

Evaluate a function

Evaluate the function $C(p) = 12p$ at $p = 3.50$.
Plug $p = 3.50$ into the expression $12p$.
Given the input $p = 3.50$, determine the output $C(p)$.

$C(3.50) = 12 \times 3.50 = \$42.00$

Evaluate the function $C(p) = 12p$ at $p = 4.00$

Evaluate the function $C(p) = 12p$ at $p = 3.50$

Solve an equation

Question: Given a cost $C(p)$, say $C(p) = \$45.00$, find the price p at which the cost is \$45.00.

Which value of p can you plug into the expression $12p$ so that $12p = 45.00$?
Given the output $C(p) = \$45.00$, what is the input p?

Solve $12p = 45.00$ for p:

Summary: If the cost to fill the tank is \$45.00, then the price per gallon is \$3.75.

Examples of units of measurement

What	Symbol & Units	Example	How Many
price of Gas	p in dollars per gallon	$p = 3.75$	\$3.75 per gallon
Gallons of Gas	$C(p)$ in dollars	$C(p) = 50$	\$50
TV Demand	d in 1000's of TVs	$d = 50$	50,000 TVs
TV Demand	d in 1000's of TVs	$d = 33$	
TV Demand	d in 1000's of TVs		43,000TVs
TV Demand	d in 1000's of TVs		43,300TVs

Definition of function and notation.

A **function** is a rule that produces a correspondence between two sets such that each element in the first set corresponds to exactly one element in the second set.

Electricity costs

Edcon power company charges its residential customers \$14.00 per month plus \$0.10 per kilowatt-hour (KWH) of electricity used. Thus, the monthly cost for electricity is a function of the number of KWHs used. In symbols, let k be the number of KWHs used in a month, and $E(k)$ be the monthly cost for electricity in dollars.

- What are the units of measurement for k and for $E(k)$?
- Write the symbolic form for the statement: The monthly cost for using 800 kilowatt-hours of electricity is \$94.00.
- Write the symbolic statement $E(660) = 80$ in words.
- Write the symbolic form for the statement: The monthly cost for using k KWHs is \$100.00.

Electricity costs

Formula for the electricity costs as a function of kilowatt-hours:

$$E(k) = 14.00 + 0.10k. \tag{1}$$

- Why 14.00?

  This is called the **fixed cost**.

- Why 0.10?

  This is called the **variable cost**.

- What is k?
- Total cost:
  (Fixed Cost) + (Variable Cost)k =

Electricity costs

$$E(k) = 14.00 + 0.10k.$$

If the customer uses 200 KWHs, find the cost.

Summary:

Electricity costs: Algebra

How many KWHs can be used if the monthly cost is \$55.00?

In this case, we know the cost but not $k =$KWHs used.

$$E(k) = 55.00.$$

Replacing $E(k)$ with the formula we get:

$$14.00 + 0.10k = 55.00.$$

Now the problem is to solve the above equation using algebra.

Summary:

Problems on electricity costs

$$E(k) = 14.00 + 0.10k$$

Translate each problem into symbolic form, solve, and write a summary.

- What is the monthly cost for using 600 KWHs?

- How many KWHs can be used if the monthly cost is \$50.00?

Price as a function of demand

The price-demand equation for selling ball point pens is

$$x = 160{,}000 - 200{,}000p,$$

where x is the number of pens (demand) that can be sold at a price of p dollars.

Express price p as a function of demand x.

Revenue as a function of demand

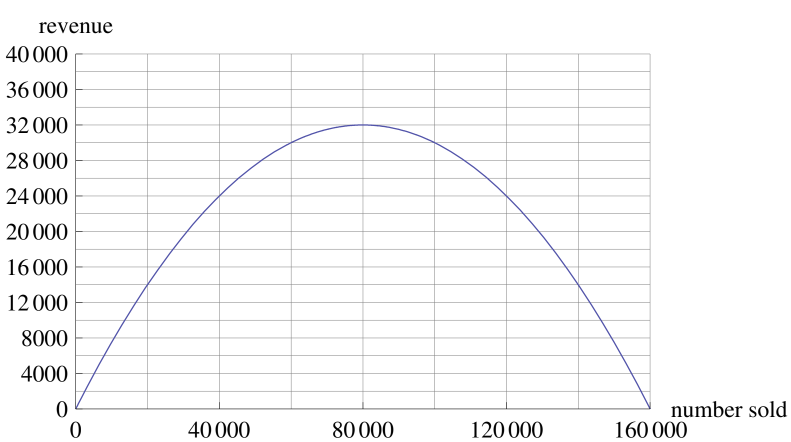
Express revenue $R(x)$ as a function of demand x. Again, revenue is demand times price:

$R(x) = xp =$

What does $R(40,000)$ mean?

Graph of revenue as a function of demand

The graph of the revenue function is a parabola:



Read $R(40,000)$ from the graph and interpret.

How draw the graph of a function

The graph of a function is a visual way to exhibit the relationship between two numerical quantities.
If we are graphing $y = f(x)$ then each point on the graph will have the form $(x, f(x))$.

For example in the previous slide we graphed $y = R(x)$ and the "highest point" on the graph was

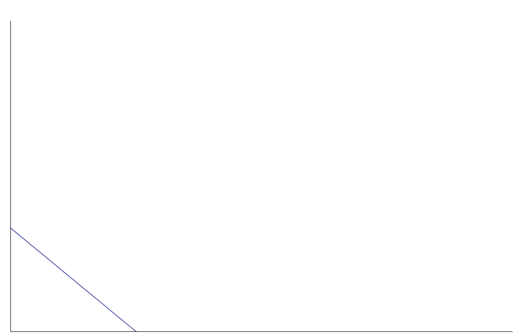
$$(80000, R(80,000)) = (80000, 32000).$$

Name some other points on that graph:

But what is the best way to make a graph? That question is hard to answer because the "best" way may depend on what it is you are trying to show with the graph. And even if the intent of the graph is clear, there still may be no single best way to draw it. Start with an easy example...

The graph of a price-demand equation

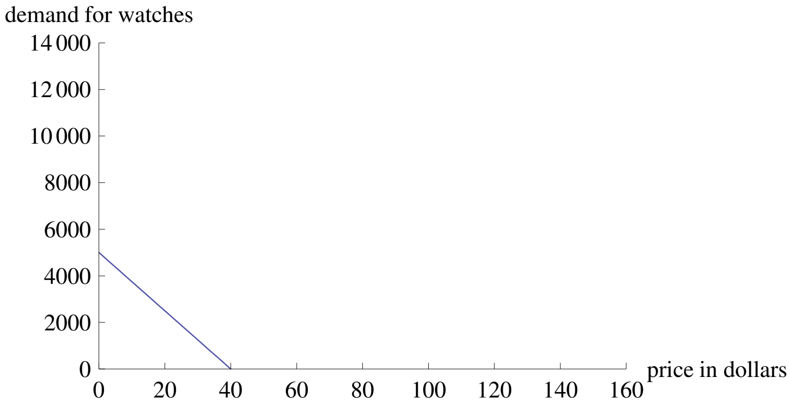
The price-demand equation for selling an inexpensive brand of wrist watch is $x = 5000 - 125p$, where x is the demand for watches at a price of p dollars.
Here is our first candidate for a graph of $x = 5000 - 125p$:



The graph above doesn't look like the ones we've seen before. What's wrong with it?

The graph of a price-demand equation

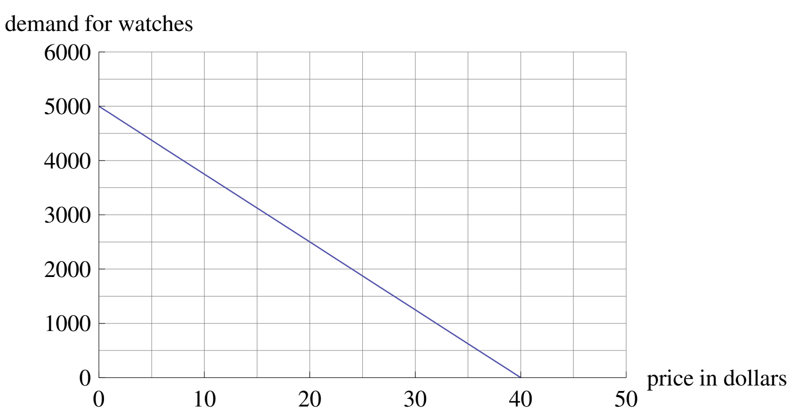
Here is another try:



There's still a problem: The important part of the graph is the line, and this line occupies only a little space in the graph.
How can this be fixed?

The graph of a price-demand equation

Here is another try:



What is good about this graph?

Guidelines for drawing a graph

- Label the axes for the graph.
- Choose an appropriate range of values for the variables.
- Select tick marks and gridlines that help the viewer read coordinates of the points on the graph.
- Write a caption that gives a very brief explanation of what the graph show.

Linear model for production costs

In the linear model for (production costs) there are fixed costs, F, and a per-item cost V.
Cost function:

$$C(x) = F + Vx$$

where $C(x)$ is the cost to produce x items. Both F and V should be apparent from the graph of the cost function.

Example: The cost to grow x rose bushes is

$$C(x) = 1000 + 10x.$$

Suppose that 1200 rose bushes is a reasonable estimate for the maximum number of rose bushes that the grower can grow and sell in Northridge.

If we draw the graph of the cost function, what would be reasonable tick marks for the x-axis?

Linear model for production costs: Rose Ex.

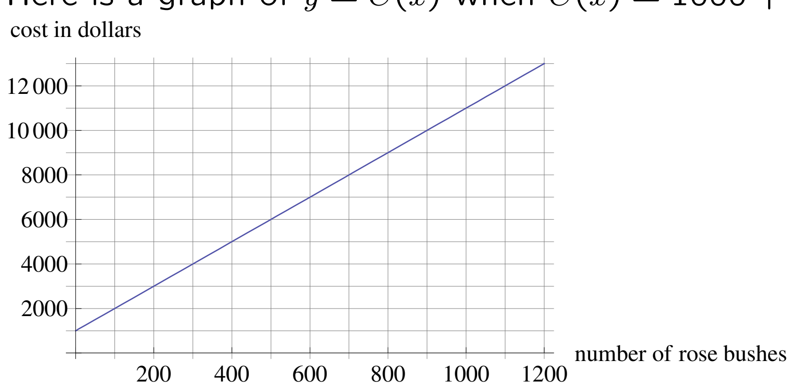
So the reasonable set of values that x can take on are between 0 and 1200.
Compute the cost to grow 1200 rose bushes.

$C(1200) = 1000 + 10(1200) = 1000 + 12000 = 13000$ dollars.

If we draw the graph of the cost function, what would be reasonable tick marks for the cost-axis? How would you label the axes?

Linear model for production costs: Rose Ex.

Here is a graph of $y = C(x)$ when $C(x) = 1000 + 10x$.



What is good/bad about this graph?

Revenue and cost on the same graph

Profit involves two things: the cost $C(x)$ to produce x items, and the revenue $R(x)$ from selling those items.
Example: Draw the plots of the cost and revenue functions for growing and selling x rose bushes:

$$C(x) = 1000 + 10x, \qquad R(x) = x(45 - 0.05x).$$

The range of values for x in the graph of the cost function for growing rose bushes is from 0 to 1200. Is that also the correct range of values for the revenue function $R(x) = x(45 - 0.05x)$?
Compute $R(1200)$:

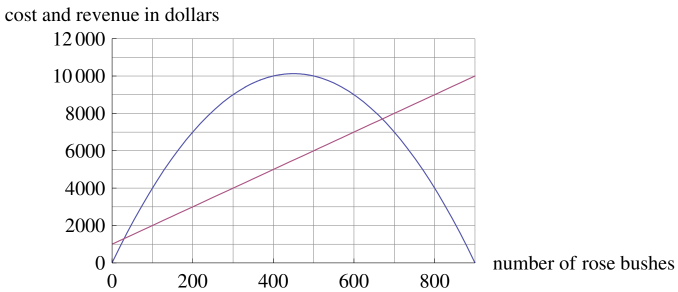
Revenue and cost on the same graph

The revenue for selling 1200 rose bushes is negative! So it doesn't make sense to sell that many rose bushes. For which values of x is revenue positive?

Revenue is the product of the number of items sold and the price at which each item sells: $R(x) = xp(x)$. So revenue is positive when both price and quantity are positive. When is that?

Revenue and cost on the same graph

So Revenue is zero when $x = 0$ or $x = 900$.
So use 0 to 900 for the possible of values on the x-axis.
A little experimentation shows that the maximum value of the revenue function $R(x)$ is a little more that 10,000. So we can restrict the values on the vertical axis to 0 to 12,000. To get:



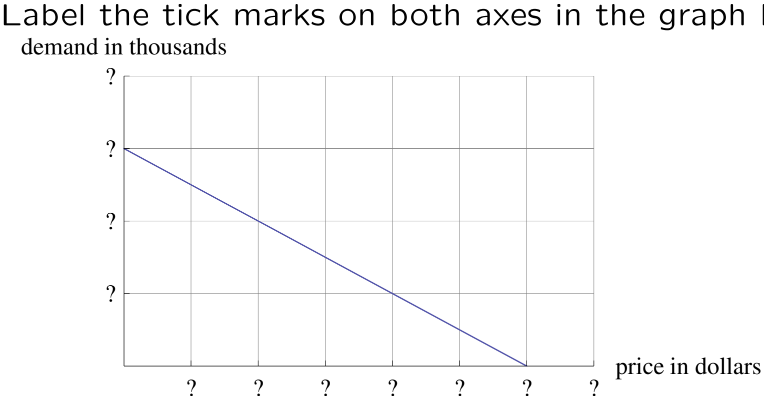
Why go all the way up to 12,000 on the vertical axis when the parabola stays well below that?

One more graphing exercise

Suppose that the price-demand equation for selling DVD players is

$$x + 2p = 300,$$

where x is the demand (in thousands) for DVD players at a price of p dollars.
Label the tick marks on both axes in the graph below.



Functions, Part 2

- Plugging into function notation
- Definition of domain and range
- Graphs
- General cost and demand functions

Function Notation

$$y = 2x - 1 \qquad f(x) = 2x - 1$$

$f(2) =$

$f(-1) =$

$f(0) =$

$f(2/3) =$

Practice

x, y equation	function
$y = -5x + 3$	$f(x) = -5x + 3$
$y = 2x^2 + 4x - .5$	$g(x) = 2x^2 + 4x - .5$

$f(-3) \quad =$ $\qquad\qquad$ $g(1) =$

$f(a) \quad =$ $\qquad\qquad$ $g(z) =$

$f(a+1) =$ $\qquad\qquad$ $=$

Last practice

$g(x) = 2x^2 + 4x - .5$

$g(a+2) =$

Definition of a domain and range of a function

A **function** is a rule that produces a correspondence between two sets of numbers such that each number in the first set (called the **domain** of the function) corresponds to exactly one number in the second set (called the **range** of the function). The domain and range are limited by both the mathematical formulas we use as well as the business context.

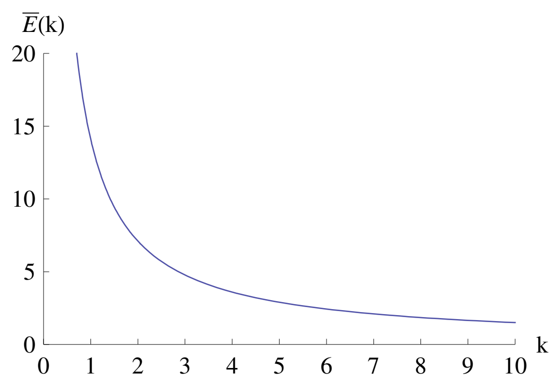
Electricity Costs

$$E(k) = 14.00 + 0.10k$$

The domain is:

The range is:

Another Business Example: $\bar{E}(k) = \frac{14.00+0.10k}{k}$

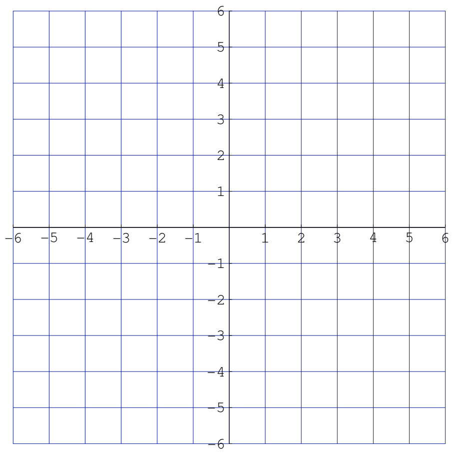


What does this expression measure?

The domain is: The range is:

Math Example:

$$y = \sqrt{x}$$



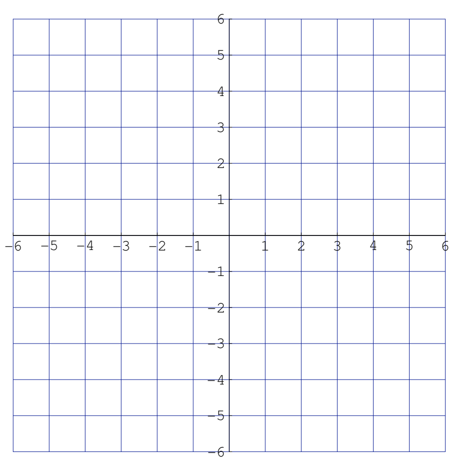
The domain is:

Inequality notation:

Interval notation:

Example Mathematical:

$$y = \frac{1}{x}$$



The domain is:

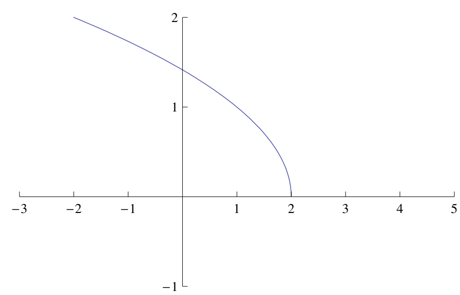
Inequality notation:

Interval notation:

Exercise: Find the Domain

Example:

$f(x) = \sqrt{2-x}$



Domain of $f(x)$:

Inequality notation:

Interval notation:

Business Example: Elasticity

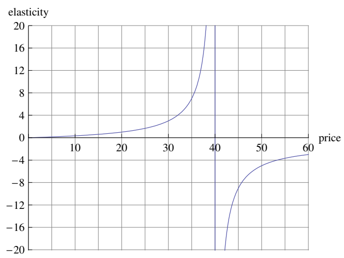
Bendie Inc. makes paperclips and sells them at a price of \$p per 1000 clips. The elasticity of demand measures how a percent change in price is translated into a percent change in demand. If Bendie's demand is measured in 1000's of paperclips and it is related to price by $d = 400 - 10p$, then we will learn later that Elasticity of demand is:

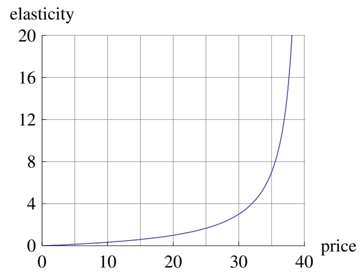
$$E(p) = \frac{p}{40-p}.$$

Mathematically what is the domain of $E(p)$?

Business Example: Elasticity

Consider two graphs of this function $E(p) = \frac{p}{40-p}$ using two different domains:



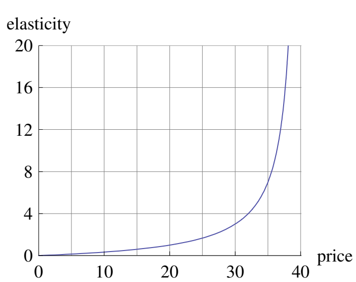


Why $E(p)$ is negative if $p > 40$?

Why is the proper domain of $E(p)$ actually $0 \le p < 40$?

Business Example: Elasticity

$$E(p) = \frac{p}{40 - p}$$



The values of $E(p)$ get very large for values of p near 40.
For example, $E(39.5) = 79$ and $E(39.9) = 399$.
The graph of $E(p)$ spikes up near $p = 40$. The vertical line at $p = 40$ is called a **vertical asymptote.**

Business Example: Profit

Suppose that Bendie's profit function is given by $P(x) = -0.1x^2 + 70x - 10,000$. The graph of Bendie's profit function is given below. Can we read the range off of the graph?