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Problem of the week 1

For m > 1, gcd(a, m)=1, the order of *a* modulo m ($ord_m a$) is the smallest positive integer k such that
$a^k \equiv 1$ (mod m). Notice that $ord_3 8 = ord_3 2 = 2$, $ord_5 8 = ord_5 3 = 4$ and $ord_{13} 8 = 4$.

So look at $N = 19\cdot 8^n + 17$ for n = 2m, we have $N \equiv 1\cdot 1 + 17 \equiv 0$ (mod 3), and hence N is not prime.

We have
$19 \equiv 1 (\text{mod} 3)$
and we note that
$8^2 = 64 \equiv 1$ so we can say that
$ord_3 8=2$
This holds since gcd(8,3)=1

So for n even the problem reduces to
$19 \bullet 8^{2n} \equiv 1 + 17 \equiv 0 (\text{mod } 3)$
So the number is not prime since 3 is a divisor!

Now for odd n, consider two cases.
For n= 3,7,11…
Look at (mod 5)
$ord_5 8=4$
m= 1,2,3…
Let n= 4m-1
$19 \bullet 8^n \equiv 4 \bullet 8^{-1} \bullet 1 + 17 \equiv 4 \bullet 2 + 17$
$= 0 (\text{mod } 5)$
and the number is divisible by 5

Doing the same thing for 1,5,9…
Let n=4m+1, but using (mod13)
$ord_{13} 8=4$
$19 \bullet 8^n \equiv 6 \bullet 8^1 + 17 \equiv 6 \bullet 8 + 17$
$= 65 \equiv 0 (\text{mod} 13)$
The number is divisible by 13.
So, I concluded the number is always divisible by 13, 5, or 3 and hence never prime!

So this exhausts the non-negative integers and I have proved it is never prime!