Problem of the Week 8, Spring 2006

Solution by George Craciun. An analytic solution looks easy. Let $ABCD$ be a rectangle with $A(0,a), B(a,c), C(c,0), D(0,0)$. The equation of $\overleftrightarrow{DB}$ is $y = (c/a)x$. The equation of $\overleftrightarrow{AP}$ is $y = c-(a/c)x$ (since $\overleftrightarrow{DB}$ and $\overleftrightarrow{AP}$ are perpendicular). Hence the coordinates of P are $x = ac^2/(a^2+c^2)$ and $y = c^3/(a^2+c^2)$. Then

$$PE = \frac{c^3}{a^2+c^2} \text{ and } PF = a - \frac{ac^2}{a^2+c^2}.$$

The relation becomes

$$\left[\frac{c^3}{(a^2+c^2)^{3/2}}\right]^{2/3} + \left[\frac{a^3+ac^2-ac^2}{(a^2+c^2)^{3/2}}\right]^{2/3} = \frac{c^2+a^2}{a^2+c^2} = 1.$$