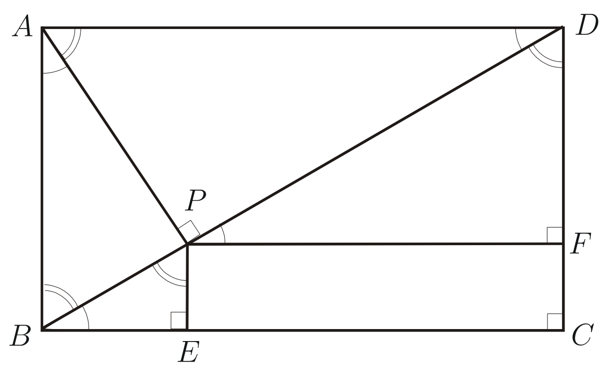
# Problem of the Week 8, Spring 2006

**Solution by organizers.** Let $AB = CD = a, BC = DA = b$, and $BD = d$. Then, since $\triangle BCD$ is a right triangle, the Pythagorean Theorem implies $a^2 + b^2 = d^2$, i.e.,

$$\left(\frac{a}{d}\right)^2 + \left(\frac{b}{d}\right)^2 = 1 \tag{1}$$

On the other hand, note that

$$\begin{aligned} \angle EBP &= \angle FPD = \angle ADP = \angle BAP \text{ and} \\ \angle BPE &= \angle PDF = \angle PBA = \angle PAD. \end{aligned}$$




Then triangles $BCD, APB$, and $BEP$ are similar and so

$$\frac{CD}{BD} = \frac{PB}{AB} = \frac{EP}{BP}.$$

The first identity gives $BP = a^2/d$ and then the second implies

$$\frac{EP}{d} = \frac{BP^2}{ad} = \frac{a^3}{d^3}. \tag{2}$$

Similarly, triangles $BCD, DPA$, and $PFD$ are similar and so

$$\frac{BC}{BD} = \frac{DP}{DA} = \frac{PF}{PD}.$$

The first identity gives $PD = b^2/d$ and then the second implies

$$\frac{PF}{d} = \frac{PD^2}{bd} = \frac{b^3}{d^3}. \tag{3}$$

Finally, equations (1), (2), and (3) imply

$$\left(\frac{EP}{d}\right)^{2/3} + \left(\frac{PF}{d}\right)^{2/3} = 1$$

which is the desired identity.