California State University Northridge

Department of Mathematics

Problem of the Week

Proposed by Bernardo Ábrego and Silvia Fernández. **April 11-18**

As every Calculus student (hopefully) knows, the formula

$$[f(x)g(x)]' = f'(x)g'(x)$$

is false in general. However it is true for some particular functions. Find two non-constant functions f and g that satisfy this formula.

Solution by Humberto Raya.

When we talk about differentiation, it becomes beneficial to talk about exponential functions and their interesting properties.
For this problem consider:

f(x)=exp[mx] and
g(x)= exp[nx] ,

for non-zero, non-one numbers m and n.
Their derivatives would be:

f'(x)=m exp[mx] and
g'(x)= n exp[nx].

By the Chain Rule we would have that

[f(x)g(x)]'=f(x)g'(x)+g(x)f'(x)
= exp[mx] n exp[nx]+ exp[nx] m exp[mx]
=(m+n)exp[(m+n)x].

Note that

f'(x)g'(x)= m exp[mx] n exp[nx]
=m n exp[(m+n)x].

In order for [f(x)g(x)]'=f'(x) g'(x) we must have that

(m+n)exp[(m+n)x]=m n exp[(m+n)x]

By dividing both sides by exp[(m+n)x] we'll see that we need:

m+n=m n.

By algebra we have that n=mn-m =(n-1)m and so m=n/(n-1).
In particular, let n=2 so that m=2.
Then **f(x)=exp[2 x]**and **g(x)=exp[2 x]** and we'll have what we wanted:

[f(x) g(x)]' = exp[2 x]2 exp[2 x]+ exp[2 x]2 exp[2 x]
= 2 exp[4 x]+ 2 exp[4 x]
= 4 exp[4 x]
=[2 exp[2 x]] [2 exp[2 x]]
= f'(x) g'(x)