

# Problem of the Week

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A digital LCD watch displays hours, minutes, and seconds in AM/PM mode. Each LCD number displayed has a certain number of *segments* turned on. For example, the number 9 has six segments, the number 1 has two segments, and at 9:02' 15'' (see figure) there are 24 segments turned on.

How many times during the day are there exactly 33 segments turned on?

**Solution by Robert Reiner.** The answer is 28 times. First, we can consider just am since pm would have exactly as many occurrences. Second, the hours can only have one of twelve possibilities:

Time|Numberofticks: 1|2, 2|5, 3|5, 4|4, 5|5, 6|6, 7|3, 8|7, 9|6, 10|8, 11|4, 12|7.

Now the minutes and seconds can both take any value from 00 to 59. The largest number of ticks the tens place can have is when the tens place is a 0, and then it would have 6 ticks. Thus the largest number of ticks the two tens places (both minutes and seconds) can have combined is 12.

Consider starting out wanting 33 ticks. Assume we use zeros for the tens' places. Removing 12 for the two 0's leaves 21.

Consider 10 o'clock. Then the 10 contributes 8 more ticks and we are left with 13. We can only get 13 by using (0,8), (6,8), (9,8), (8,0), (8,6), (8,9) where  $(x, y)$  means put  $x$  for the ones of the minutes and put  $y$  for the ones of the seconds. All other combinations would yield too small a value (except (8,8) which would give us 34 total). Also, if we want something other than 0 in the tens places, we can use (0,5), (0,2),(0,3),(0,5),(2,0),(3,0),(5,0) for tens places as long as we use (8,8) for ones places. All other values would yield too few ticks. Thus 10 o'clock has 12 possibilities:

10:00:08	10:08:00	10:28:08	10:08:28
10:06:08	10:08:06	10:38:08	10:08:38
10:09:08	10:08:09	10:58:08	10:08:58

Consider 12 o'clock. The 12 contributes 7 more ticks, and we are left with needing 14 ticks. The only way to achieve this is with (8,8) for the one's positions. Thus 12

o'clock contributes 1 possibility:

12:08:08

Similarly 8 o'clock contributes 7 more ticks leaving 14 remaining to be found which forces (8,8) for the one's positions. Thus 8 o'clock contributes 1 possibility:

8:08:08

All other hours possibilities yeild too few ticks too be made up by any combination of minutes and second.

Thus the total for am in  $12 + 1 + 1 = 14$ , so the total for the day is  $14(2) = 28$ .