

Problem of the Week

Proposed by Bernardo Ábrego and Silvia Fernández. **March 28-April 4**

Jack wants to write a sum of numbers equal to 100 by using each of the digits 1 to 9 exactly once. His best try so far is:

$$4 + 6 + 9 + 18 + 25 + 37 = 99.$$

Show that, no matter how hard he tries, he will never succeed.

Solution by Barbara Flakowski. We begin with the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9. Some of them will be multiplied by ten and others by 1. Then the sum of the resulting numbers will be taken. Of the digits five are odd. For the final sum to be even either 0, 2 or 4 of the digits must be multiplied by 1 while the others are multiplied by 10. The sum of the five numbers must be less than 100. The following cases fit these specifications.

$$7 \times 10 + (1 + 3 + 5 + 9) \times 1 = 88$$

$$5 \times 10 + (1 + 3 + 7 + 9) \times 1 = 70$$

$$3 \times 10 + (1 + 5 + 7 + 9) \times 1 = 52$$

$$1 \times 10 + (3 + 5 + 7 + 9) \times 1 = 34$$

The even digits must also be multiplied by 1 or 10 and the resulting numbers summed. The resulting sums must be less than 100. The sums that qualify are:

$$(2 + 4 + 6 + 8) \times 1 = 20$$

$$2 \times 10 + (4 + 6 + 8) = 38$$

$$4 \times 10 + (2 + 6 + 8) = 56$$

$$6 \times 10 + (2 + 4 + 8) = 74$$

$$8 \times 10 + (2 + 4 + 6) = 92$$

Now we have to check if any of the sums from the odd digits added to any of the sums from the even digits added together equal 100. None of them do.