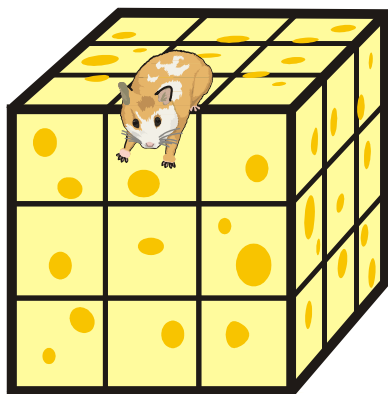


Problem of the Week

Proposed by Bernardo Ábrego and Silvia Fernández.

February 28-March 7



A $3 \times 3 \times 3$ cube of cheese is divided into twenty seven $1 \times 1 \times 1$ small cubes. A hamster eats one small cube every day and an *adjacent* small cube (sharing a face) the next day. Can the hamster eat the *center* small cube on the last day? Explain your answer.

Deadline: March 7, 2005 before 9:00 PM.

Look for the “Problem of the Week” every Monday in the Daily Sundial (Daily Spotlight section) or in our web site www.csun.edu/math/probweek

Solution by Barbara Falkovski. The hamster has a total of 27 small cubes. One is in the *center* of the larger cube, eight are at *corners*, twelve are on the *edges* (between corners), and six are in the *centers of faces*. After eating the center of a face the hamster may eat an edge or the center of the cube. After eating a corner the hamster must eat an edge. After eating an edge the hamster must eat either the center of a face or a corner.

If the hamster decides to eat the middle of the cube last, it will always eat an edge after eating either a corner or a center of a face. It will be forced to alternate between eating an edge and eating either a corner or a face center. There are twelve edges and the sum of corners and face centers is fourteen so there will be two corners or face centers left after all the edges have been eaten. If there is at least one corner left it will have no adjacent small cube, so the two remaining pieces must be centers of faces. They are necessarily adjacent to the center of the large cube, which separates them from each other. Because of this the hamster cannot eat the center of the cube last.