Problem of the Week. **April 19-26**

Proposed by Bernardo Ábrego and Silvia Fernández.

Two players A and B alternate turns during a game as follows: Player A starts by calling a whole number between 1 and 10. Each turn a player calls a whole number larger than the previous by at most 10. The player who calls 100 wins. For example, a game can start as A calls 3, B calls 12, A calls 22, B calls 24, A calls 25, etc.
Give a winning strategy for player A. Explain why this strategy always works.

Solution by Prashant Saraswat. If A is to say 100, then, since he can only say integers that are 10 or more greater than the last number B, said B must say some number from 90 to 99 inclusive immediately before this. B must be put into a position where he is forced to say a number from 90 to 99 inclusive. This is done by having A say 89—the lowest number B can then say is one more, 90, and the highest is 10 more, 99. We have the same problem we started with, getting A to say a particular number. If A is to say 89, then B must have said a number from 79 to 88 inclusive immediately beforehand, meaning that A must say 78 in order to force this upon B. The pattern continues, with A having to say the number $100-11n$ when he is n turns away from winning (that is, saying 100). Therefore he must start with $100-11\,(9)=1$. B is then forced to say a number from 2 to 11 inclusive; no matter what he picks, A will be able to say 12 on the next turn, and 23 on the next (B having been forced to say a number from 13-22), and so forth with A always being able to say $1+11(n-1)$ on his nth turn, until on his 10th turn he says 100. He does not need to pay any regard to what numbers B says, if he simply says the numbers 1, 12, 23, 34, 45, 56, 67, 78, 89, and 100 in that order he will win no matter what B does.