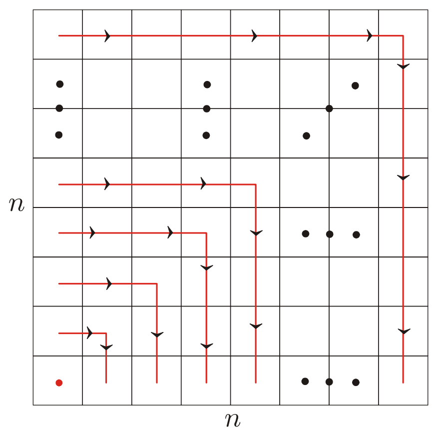
Problem of the Week #1 March 8-15, 2004

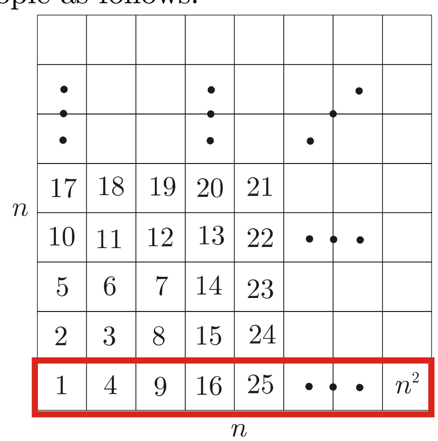
Proposed by Bernardo Ábrego and Silvia Fernández.

Let n be a positive integer. A group of n^2 people is divided into smaller groups according to the following procedure: Each person is assigned a different number from 1 to n^2. The first group consists of all people whose number is a perfect square. After removing this group, the remaining people are renumbered starting from 1 again. The second group consists of all people whose new number is a perfect square. The process of renumbering the people and removing the group of perfect squares is repeated until one person is left. This person is the only member of the last group. How many groups were formed?

Solution. We number the people as follows:

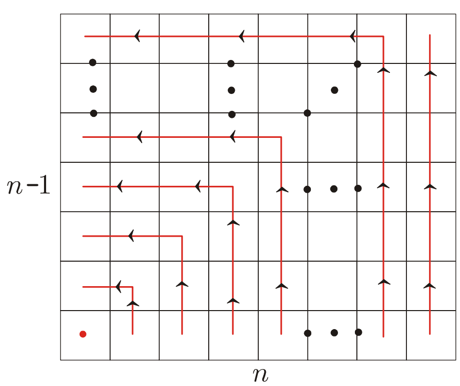


$n \times n$ square

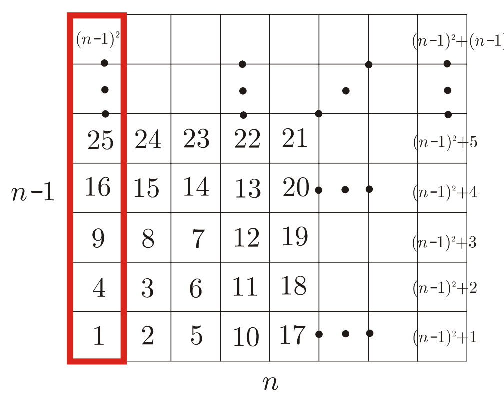


Remove the first group(last row)

Renumber the remaining people as follows:



$(n-1) \times n$ rectangle



Remove the second group(first column)
to get a $(n-1) \times (n-1)$ square

After removing $n-1$ rows and $n-1$ columns we end up with a 1×1 square which represents the only person in the last group. Thus we formed a total of $(n-1)+(n-1)+1 = 2n-1$ groups.

[We thank Prof. Ann Watkins for this idea.]