Problem of the Week #7

Original Problem: Prove that if nine points are chosen in the interior of a unit square, then three of them form a triangle with area at most 1/8. (Note: If three points are on the same line, we say that the triangle they form has area zero).

Further questions for future research projects.

1. It is known that 6 points are enough to guarantee a triangle with area 1/8. A proof of this would be an excellent research project. The following is a figure with six points in the unit square where all triangles have area at least 1/8.



2. The problem of maximizing the area of the smallest triangle determined by 7 points in the square is open. The best lower bound is 0.0838 by Comellas and Yebra (2001). It would be an excellent project to find an upper bound close to this number

The problem for general n is called Heilbronn's triangle problem. You can read more information about it here: http://www.stetson.edu/~efriedma/heilbronn/