Problem of the Week #5 **April 12-19, 2004**

Original Problem: Let $P(x) = x^4 + ax^3 - bx^2 + cx + 1$ be a polynomial with real coefficients. Prove that if $|a+c| < b-2$ then P has four different real roots (that is, there are four different real values of x for which $P(x) = 0$).

Further questions for future research projects.

If we are giving the polynomial $P(x) = x^4 + ax^3 - bx^2 + cx + d = 0$, how can we check if a set of inequalities would force the polynomial to have four real roots. For example the original problem had the inequalities $a + c < b - 2$ and $-b + 2 < a + c$. To simplify the problem we can start with only two linear inequalities. Here are some precise examples of such questions.

1. If $25a + c < 5b - 126$ and $9 - 2b < 4a + c$ does it follow that $P(x)$ has four real roots?
2. If $9a + c < 3b - 27$ and $64 - 4b < 16a + c$ does it follow that $P(x)$ has four real roots?

If the answers is no then you should be able to construct an explicit $P(x)$ with less than four roots.