Problem of the Week #1 — March 8-15, 2004

Original Problem: Let n be a positive integer. A group of n^2 people is divided into smaller groups according to the following procedure: Each person is assigned a different number from 1 to n^2. The first group consists of all people whose number is a perfect square. After removing this group, the remaining people are renumbered starting from 1 again. The second group consists of all people whose new number is a perfect square. The process of renumbering the people and removing the group of perfect squares is repeated until one person is left. This person is the only member of the last group. How many groups were formed?

Further questions for future research projects.

1. You can ask the same question but now starting with an arbitrary number of people, say m. In other words, in the original problem $m = n^2$ is a perfect square, but now m can be *any positive integer*. What happens now? You can introduce new notation. For example, you can define the function $f(m)$ to be the number of groups formed following the rules of the problem (the group formed at each stage consists of all people whose number is a perfect square). What can you say about the function $f(m)$?

   For instance, from the solution to the original problem we know that $f(n^2) = 2n - 1$.

2. Ask a similar question, but now start with n^3 people. After renumbering the remaining people at each stage, form a group with all the people whose number is a *perfect cube*. How many groups are formed this time?

3. If you still want more questions, replace the "squares" in the original problem by *any* k^{th}*-power*. (the original problem uses $k = 2$ and question #2 uses $k = 3$).