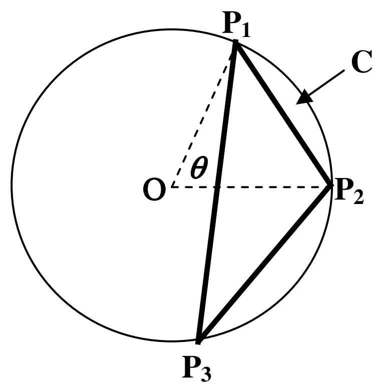
The expected area of the triangle can be found by first determining the expected area *outside* of the triangle and then subtracting it from the area of the circle. That is, $E(\Delta) = \pi - 3E(\mathbf{A})$, where **A** is the area of any circular segment **C** contained between a side of the triangle and the circle:



We can assume that the three randomly selected points $\mathbf{P_1}, \mathbf{P_2}, \mathbf{P_3}$ are distinct, as this is a probability 1 event. Let $\theta \in (0, \pi]$ be the (smaller) central angle between any two points, say $\mathbf{P_1}$ and $\mathbf{P_2}$, as shown above.

If $\mathbf{P_3}$ falls within the *longer* arc between $\mathbf{P_1}$ and $\mathbf{P_2}$, as shown, then

$$\mathbf{A} = \text{area of the } \textit{sector } \mathbf{OP_1P_2} - \text{area of the } \textit{triangle } \mathbf{OP_1P_2} = S(\theta) = \frac{\theta}{2} - \frac{\sin\theta}{2}.$$

This case occurs with probability $1 - \frac{\theta}{2\pi}$, since the shorter arc length between $\mathbf{P_1}$ and $\mathbf{P_2}$ is θ.

For the case when $\mathbf{P_3}$ is within the *shorter* arc between $\mathbf{P_1}$ and $\mathbf{P_2}$, which occurs with probability $\frac{\theta}{2\pi}$, the area **A** will be that of the complement of the region **C** shown above, or $\pi - S(\theta)$.

As the probability density of θ is $f(\theta) = \frac{1}{\pi}$, we have

$$E(\mathbf{A}) = \int_0^{\pi} \left[\left(1 - \frac{\theta}{2\pi}\right) S(\theta) + \frac{\theta}{2\pi}(\pi - S(\theta)) \right] \frac{1}{\theta}\, d\theta$$

$$= \int_0^{\pi} \left[\frac{\theta \sin\theta}{2\pi} - \frac{1}{2}\sin\theta - \frac{\theta^2}{2\pi} + \theta \right] d\theta = \frac{\pi^2}{3} - \frac{1}{2}.$$

We thus obtain $E(\Delta) = \pi - 3E(\mathbf{A}) = \boxed{\frac{3}{2\pi}}$. The expected area of a random triangle is thus only about 15% of the area of the circle.