Pick the three random points as follows: First choose two random diameters $D_1$ and $D_2$, then randomly pick an end of each diameter as the points $P_1$ and $P_2$. Then pick the third random point $P_3$. Before the process begins it is equally likely for $P_3$ to fall into any one of the four arcs that $D_1$ and $D_2$ will partition the circumference of the circle into. In one of these cases (Figure 1), the resulting triangle $\mathbf{P_1P_2P_3}$ will contain the center of the circle, **O** (see Figure 2), and in the other three cases it will not (see Figure 3 for one such case).

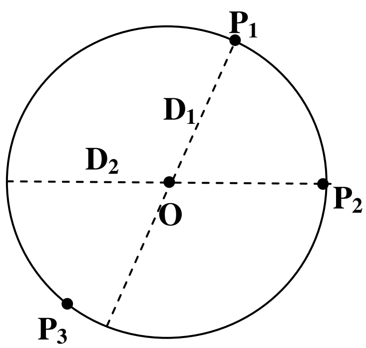


**Figure 1**

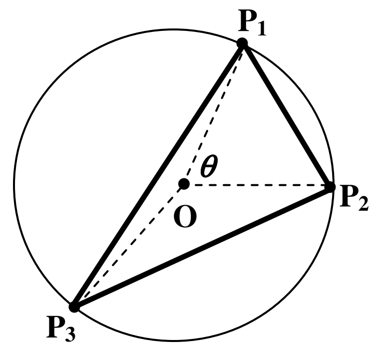


**Figure 2**

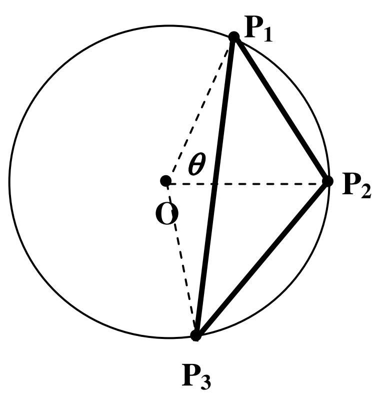


**Figure 3**

Let **A** be the area of the triangle $\mathbf{P_1P_2P_3}$. We wish to find E(**A**). Let $A_{12}$, $A_{13}$, $A_{23}$ be the areas of the triangles $\mathbf{OP_1P_2}$, $\mathbf{OP_1P_3}$, and $\mathbf{OP_2P_3}$, respectively. Each of these three areas has the same distribution and hence the same expected value. Letting $\theta$ be the angle $\angle \mathbf{P_1OP_2}$, we have

$$E(A_{12}) \text{ (say)} = E\left(\frac{1}{2}\sin\theta\right) = \int_0^{\pi}\left(\frac{1}{2}\sin\theta\right)\frac{1}{\pi}d\theta \quad (\text{since } \theta \text{ is uniformly distributed on } [0,\pi]) = \frac{1}{\pi}.$$

We can see that in the first case (Figure 2), $A = A_{12} + A_{13} + A_{23}$, while in the other three cases, $A = A_{12} + A_{23} - A_{13}$ (the case shown in Figure 3) or $A = A_{12} + A_{13} - A_{23}$ or $A = A_{13} + A_{23} - A_{12}$. Therefore

$$E(\mathbf{A}) = (\tfrac{1}{4}) \times E(A_{12} + A_{13} + A_{23}) + (\tfrac{3}{4}) \times E(A_{12} + A_{23} - A_{13}) = (\tfrac{1}{4}) \times E(3A_{12}) + (\tfrac{3}{4}) \times E(A_{12})$$

$$= \frac{3}{2}\, E(A_{12}) = \frac{3}{2\pi}.$$