Problem of the Week 8, Fall 2008

Find nine different prime numbers $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8$, and p_9, (see the figure) so that the sum of the three entries in any column, row, or diagonal is also a prime number.

p_1	p_2	p_3
p_4	p_5	p_6
p_7	p_8	p_9

The first solution with smallest sum $p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9$ wins.

Solution by organizers. Note that 2 cannot be used because the sum of two odd primes and 2 is an even number greater than 2 and thus not a prime. A possible solution is ($p_1 = 17, p_2 = 3, p_3 = 23, p_4 = 7, p_5 = 5, p_6 = 29, p_7 = 13, p_8 = 11$, and $p_9 = 19$)

17	3	23
7	5	29
13	11	19

Because the first nine odd primes are $3, 5, 7, 11, 13, 17, 19, 23$, and 29, this is a solution with smallest possible sum $p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 = 127$.

Other solutions.

3	5	23
7	11	19
13	17	29

by Virgilio Cerna
Total sum 127

3	5	23
7	11	19
13	31	29

by Virgilio Cerna
Total sum 141

17	23	13
19	3	7
31	5	11

by Martha J. Dawiczyk
Total sum 129

3	5	29
7	11	13
19	37	17

by Chuck Goodman
Total sum 141

3	5	11
17	7	13
23	29	19

by David Roberts
Total sum 127

3	5	11
7	13	17
19	23	31

by Edouard A. Tchertchian
Total sum 129

Project. As you can see, there are many solutions. It would be interesting to know how many different solutions have a total sum of 127. In other words, all solutions using the primes $3, 5, 7, 11, 13$, 17, $19, 23$, and 29 exactly once. It can proved for example that no such solution has the number 3 in the center.