

Problem of the Week 4, Fall 2008

There are 50 people in a party. Each person knows at least one other person. At some point a celebrity enters the party. Everybody knows the celebrity but the celebrity knows no one. You are new in town and get to the party after the celebrity. You don't know anybody, including the celebrity.

Trying to hide the fact that you don't read the tabloids, you want to figure out who the celebrity is by asking questions of the form "Do you know this person?" ("This person" could be anyone you choose from the party.) How can you do this by asking at most 50 questions?

Solution by organizers. There are 51 people in the party (without counting yourself). We describe an algorithm that takes exactly 50 steps to find the celebrity. Each step consists of asking the question "Do you know this person?" once and deleting one person who is not the celebrity from the list of guests in the party. At the end, there is only one person on the list. This person is the celebrity.

Step 1. Pick any two people at random, call them p_0 and p_1 . Ask p_1 "Do you know this person?" pointing at p_0 . If p_1 answers *no*, then p_0 is not the celebrity (everyone knows the celebrity) and should be deleted from the list of guests. If p_1 answers *yes*, then p_1 is not the celebrity (the celebrity knows no one) and should be eliminated from the list.

Step k . ($2 \leq k \leq 50$) Suppose q is the person you decide to keep in step $k - 1$. Pick another person at random who is still in your list, say p_k . (That is, a person to whom you have not asked questions.) Ask p_k "Do you know this person?" pointing at q . As in step 1, if p_k answers *no*, then q is not the celebrity (everyone knows the celebrity) and should be deleted from the list of guests. If p_k answers *yes*, then p_k is not the celebrity (the celebrity knows no one) and should be eliminated from the list.