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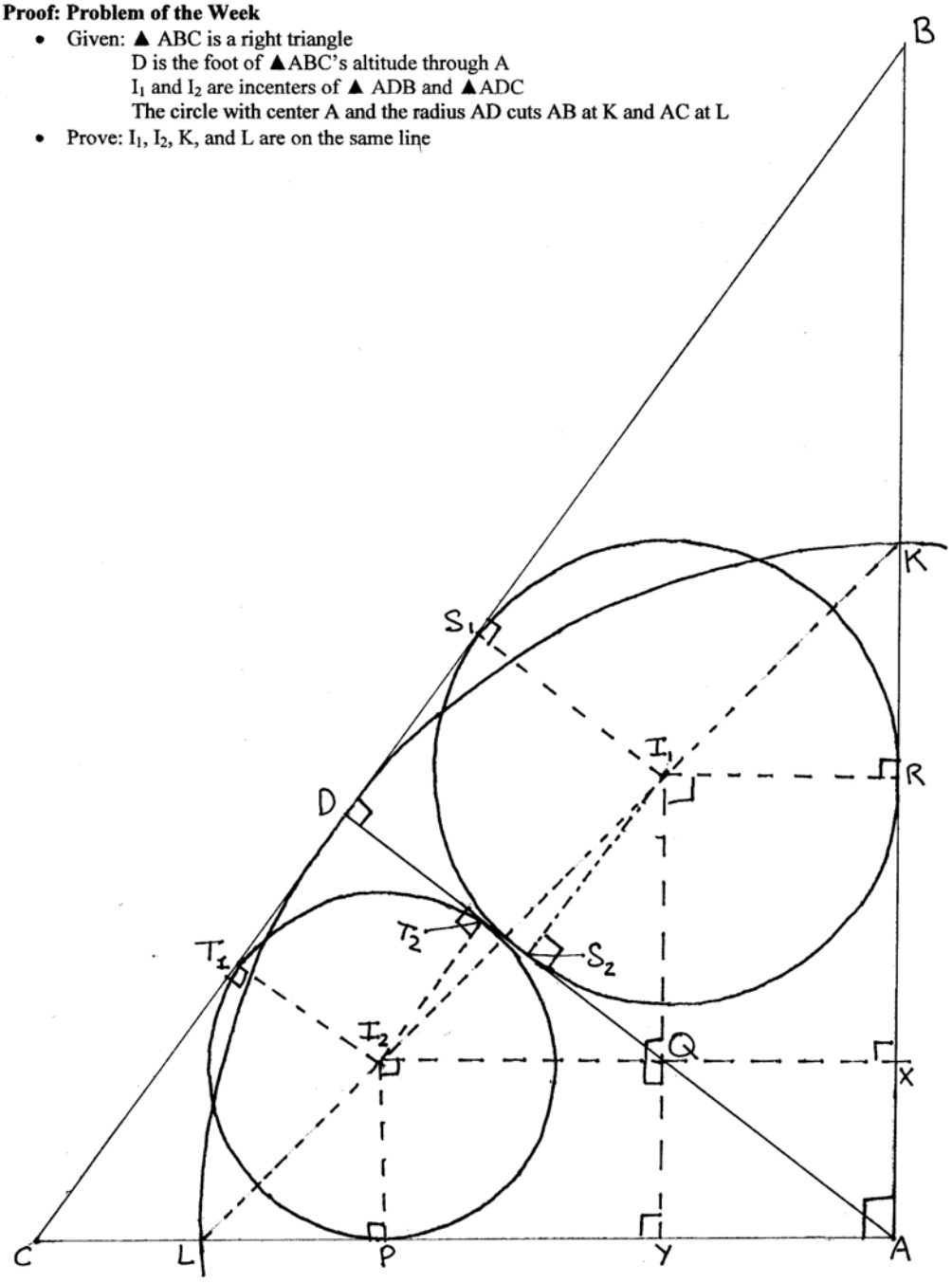
Proof: Problem of the Week

- Given: ▲ ABC is a right triangle
  D is the foot of ▲ABC's altitude through A
  I_1 and I_2 are incenters of ▲ ADB and ▲ADC
  The circle with center A and the radius AD cuts AB at K and AC at L
- Prove: I_1, I_2, K, and L are on the same line



Statement	Reason
1. I_2 and I_1 are centers of circles that are inscribed in triangles ADC and ADB respectively	1. definition of incenters
2. Let P be the point where circle I_2 intersects AC	2-3. Inscribed circles have one tangent to each side of the triangle
3. Let R be the point where circle I_1 intersects AB	
4. Construct a line that is parallel to AB, that passes through I_1, and that intersects AC at Y	4. Construction
5. Construct a line that is parallel to AC, that passes through I_2, and that intersects AB at X	5. Construction
6. Let Q be the point where I_1Y and I_2X intersect	6. the lines intersect
7. Angle $CYI_1 = CAB$	7. Corresponding angles are equal
8. Measure of angle $CYI_1 = 90$ *degrees*	8. Substitution
9. Angle $CYI_1 = I_2QI_1$	9. Corresponding angles are equal
10. Measure of Angle $I_2QI_1 = 90$ *degrees*	10. Substitution
11. Measure of Angle $I_2XB = 90$ *degrees*	11-13. Corresponding Angles and substitution
12. Measure of Angle $PI_2X = 90$ *degrees*	
13. Measure of Angle $RI_1Y = 90$ *degrees*	
14. $PAXI_2$ is a rectangle	14-17. if all the interior angles of a quadrilateral are right angles, then the quadrilateral is a rectangle
15. $RAYI_1$ is a rectangle	
16. QI_1RX is a rectangle	
17. QI_2PY is a rectangle	
18. I_1R is the radius of Circle I_1, and I_2P is the radius of Circle I_2	18. Length from center of circle to tangent = radius
19. $I_2Q = I_2X - QX$	19. Addition prop. of segments
20. $QX = I_1R$	20. def. of rectangle
21. I_1R = *Radius of circle* I_1	21. Reason #18
22. $I_2X = AP$	22. def. of rectangle
23. $I_2Q = AP$- *Radius of circle* I_1	23. substitution for step 18
24. $I_1Q = I_1Y - QY$	24. Reason # 18
25. $QY = I_2P$	25. Reason #19
26. I_2P = *Radius of circle* I_2	26. Reason #20

27. $I_1Y = AR$	27. Reason #21
28. $I_1Q = AR$ – *Radius of Circle* I_2	28. substitution from step 23
29. *Radius* $I_1 = -½*(AB-DB-DA)$; *Radius* $I_2 = -½*(AC-DC-DA)$	29. The radius of circle inscribed in right triangle theorem
30. Let T_1 and T_2 be the points where Circle I_2 is tangent to sides CD and AD, respectfully	30-31. Reason # 2-3
31. Let S_1 and S_2 be the points where Circle I_1 is tangent to sides BD and AD, respectfully	
32. $I_2T_1DT_2$ and $I_1S_1DS_2$ are squares	32. If a circle is inscribed in a right triangle, the quadrilateral formed from it radii, the sectors of the two legs, and the main right angle is a square
33. I_2T_1 is the radius of circle I_2, I_1S_1 is the radius of circle I_1	33. Reason #18
34. $T_2D = I_2T_1$, $S_2D = I_1S_1$	34. def. of square
35. T_2D and S_2D are the radii of circles I_2 and I_1	35. substitution
36. $AP = AT_2$, $AR = AS_2$	36. Tangents to a circle from an external point are equal
37. $AT_2 = AD - T_2D$, $AS_2 = AD - S_2D$	37. Reason #19
38. $AT_2 = AD$ – *Radius of* I_2, $AS_2 = AD$ – *Radius of* I_1	38. substitution from step #35
39. $AP = AD$ – *Radius of* I_2, $AR = AD$ – *Radius of* I_1	39. substitution from step #36
40. $AP = AD - [-½*(AC-DC-DA)]$, $AR = AD - [-½*(AB-DB-DA)]$	40. substitution from step #29
41. $AP = ½AD+½AC-½CD$, $AR = ½AD+½AB-½BD$	41. Arithmetic
42. $I_2Q = AP$- *Radius of circle* I_1, $I_1Q = AR$ – *Radius of Circle* I_2	42. Reason #19
43. $I_2Q = [½AD+½AC-½CD] - [-½*(AB-DB-DA)]$, $I_1Q = [½AD+½AB-½BD] - [-½*(AC-DC-DA)]$	43. Substitution from steps #41 and #29
44. $I_2Q = ½AC-½CD+½AB-½DB$ $I_1Q = ½AC-½CD+½AB-½DB$	44. Arithmetic
45. $I_2Q = I_1Q$	45. Substitution
46. I_2QI_1 *is a right isosceles triangle*	46. a triangle with two equal side and a right angle is a right isosceles triangle
47. Angle QI_2I_1 *is 45 degrees,* Angle QI_1I_2 *is 45 degrees*	47. definition of right isosceles triangle

48. $RK = AK-AR$, $PL = AL-AP$	48. Reason #19
49. $AK = AL = AD$	49. Given
50. $AR = AS_2 = AD - S_2D$, $AP = AT_2 = AD - T_2D$	50. Reasons #19 and #36
51. $RK = AD-[AD-S_2D]$, $PL = AD-[AD-T_2D]$	51. Substitution from step #48
52. $RK=S_2D$, $PL=T_2D$	52. arithmetic
53. RK = *Radius of circle* I_1, PL = *Radius of circle* I_2	53. substitution from step #34
54. I_1R = *Radius of Circle* I_1, I_2P = *Radius of Circle* I_2	54. Reason #18
55. $RK=I_1R$, $PL=I_2P$	55. Substitution
56. *Triangles* I_1RK and I_2PL *are right isosceles triangles*	56. Reason #46
57. Angles I_2LP, LI_2P, KI_1R, *and* I_1KR *are 45 degrees each.*	57. Reason #47
58. Angles PI_2Q *and* RI_1Q are right angles	58. definition of rectangle
59. Angles LI_2I_1 and I_2I_1K are straight angles.	59. Angles whose measure is 180 degrees is a straight angle
60. L, I_2, I_1, *and* K *are on the same line.*	60. def. of a straight angle