Problem of the Week 1, Fall 2006

Solution by organizers. Note that we are interested in the *order* in which the spider gets its socks and shoes on, rather than in its *outfit*.

Label its feet F1, F2, F3, F4, F5, F6, F7, F8. Note that any arrangement of the numbers in the chain 1122334455667788 corresponds to a different order that the spider can use to get dressed (and any order can be represented by an arrangement) where the first appearance of a number corresponds to putting on a sock and the second to putting on a shoe on that foot. For example, the arrangement

$$1863281752365447$$

corresponds to sock on F1, sock on F8, sock on F6, sock on F3, sock on F2, shoe on F8, shoe on F1, sock on F7, sock on F5, shoe on F2, shoe on F3, shoe on F6, shoe on F5, sock on F4, shoe on F4, and shoe on F7.

Then there are $\dfrac{16!}{2^8} = 81\,729\,648\,000$ different orders: There are 16! ways to permute the 16 digits. Since each of the eight digits appears twice, every permutation is counted 2^8 times. Thus we must divide 16! by 2^8 to account for all the permutations that were multiply counted.