Problem of the Week 11, Fall 2005

Solution by Jason Hughes (edited). Since $y = x^2$ is a twice differentiable function, there exists a curvature κ at the point (x, y) given by

$$\kappa = \frac{|y''|}{\left(1 + (y')^2\right)^{3/2}}.$$

The inverse of the curvature indicates the radius of curvature. The point with the largest curvature, and thus with the smallest radius of curvature in the parabola $y = x^2$, is the point $(0, 0)$. Differentiating $y = x^2$ we get $y' = 2x$, and $y'' = 2$. Then

$$\kappa(0, 0) = \frac{2}{\left(1 + (0)^2\right)^{3/2}} = 2.$$

Thus the required radius of curvature is $1/\kappa = 1/2$.