Problem of the Week 1, Fall 2005

Solution to by Lucas Lembrik (edited). Define a layer as the diagonal edges inbetween the horizontal edges. If you consider the top layer of the triangle there are only two ways to get to the top. Namely straight up the left side, or over one spot and then up the right side.

Next consider just the second to top layer. To get from the bottom of this layer to the bottom of the top layer there are four different ways up. So to get from the bottom of this two layered triangle to the top there are 4*2 or 8 different possibilities. These possibilities correspond to the product of the ammount of diagonal edges in each layer. So to get from the bottom of the i^{th}layer to the bottom of the $(i-1)^{\text{th}}$ layer there are $2i$ different possibilities, because there are $2i$ diagonal edges in this layer.

Continuing this pattern for all six layers of the triangle there are $12 \cdot 10 \cdot 8 \cdot 6 \cdot 4 \cdot 2$ different possibilities or $2^6(6!) = 46080$ different ways to get from the bottom of the 6-layered triangle to the top. Note that, once in the bottom of the last layer, the way to reach B is fixed.