California State University
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Department of
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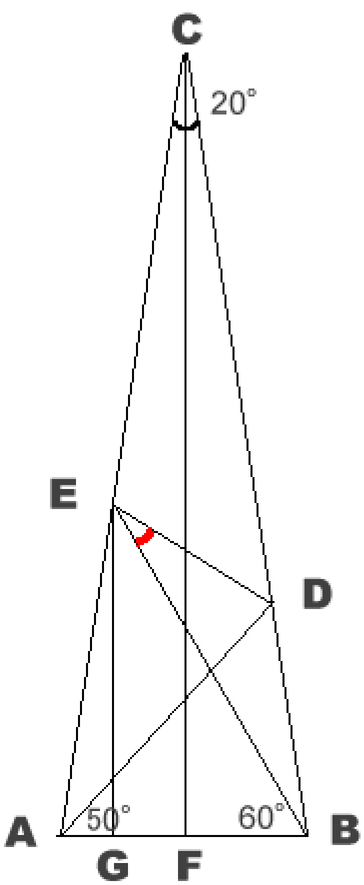
Problem of the Week

Proposed by Miroslav Tanushev. **November 1-8**

Let ABC be an isosceles triangle with $BC = CA$ and $\angle BCA = 20°$. Points D and E are on the sides BC and CA, respectively, and they satisfy that $\angle DAB = 50°$ and $\angle ABE = 60°$. Find, with proof, the exact value of the angle $\angle DEB$.

Solution by: Yuko Takagi



* This figure may not be accurate.

Objectives: To prove ΔCAD and ΔBED are similar triangles by comparing the ratio of the lengths of the sides, and find out the value of the angle $\angle$DEB.

[Step 1]
From the given information, the followings are determined.
$\angle ADB = 50^\circ$
$\angle CBE = 20^\circ$
$\angle DAC = 30^\circ$

[Step 2]
Let the lengths of CA and CB = 1.
Let the lengths of AB = a.

From the given information and Step 1, ΔBAD is an isosceles triangle.
Therefore, the length of DB = a.

[Step 3]
Let the length of CE equal to x.
Since ΔCEB is an isosceles triangle(see Step 1), the length of BE = x.
Now, I describe x in terms of a.

[Step 4]
Draw a perpendicular line from C to the line AB. Label it F.
Draw a perpendicular line from E to the line AB. Label it G..

FG : GA = CE : EA = x : 1-x → (1)

[Step 5]
Since ΔABC is an isosceles triangle, F equally divides the line AB.
ΔBEG is a half of a equilateral triangle, therefore the length of BG is half of the length of BE.

FG : GA = BG-BF : AB-BG = x/2 – a/2 : a – x/2 → (2)

(1) and (2) are the same ratio. Now solve this for x.

$x : 1-x = x/2 - a/2 : a - x/2$

$x = a/(1-a)$ → (3)

[Step 6]

With those being solved, the ratio of BE : BD will be

$BE : BD = a/(1-a) : a = 1 : 1-a$ → (4)

which is the same as CA : CD.

Since the ratio of lengths of two sides are the same and the angles between the sides are the same as well, ΔCAD and ΔBED are similar triangles.

Therefore, $\angle DEB = \angle DAC = 30^\circ$ <u>$\angle DEB = 30^\circ$</u>