California State University
Northridge

Department of Mathematics

# Problem of the Week

Proposed by Bernardo Ábrego and Silvia Fernández. **October 4-11**

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Consider a lamp fixture with 1000 lights in a row, and 1000 CSUN students. Each light can either be on or off, and it has a button that when pressed switches the status of the light (from 'on' to 'off' or from 'off' to 'on').

At the beginning all lights are off, then the 1000 students proceed to change the lights in the following way: Student 1 pushes every button, Student 2 pushes every other button starting at 2, that is buttons 2,4,6,8,etc. Student 3 pushes every third button starting at 3, that is buttons 3,6,9,etc. The rest of the students follow the same pattern. For example, Student 100 pushes buttons 100,200,300, etc. The process ends with Student 1000 who simply pushes button 1000.

How many lights are turned on in the end? Justify your answer.

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**Solution by Chris Dungan.** The problem regarding the switching of 1,000 lights based on their assigned number would, if executed, result in each light being switched once for each factor its number has. Since every light starts in the off state, only lights assigned numbers with an odd number of factors will be on at the end. All perfect squares have matched pairs with one pair of [equal] factors (e.g., for 144: 1 & 144, 2 & 72, 3 & 48, 4 & 36, 6 & 24, 8 & 18, 9 & 16 and 12 & 12) and all other numbers cannot have such matched pairs. Therefore, all perfect squares have an odd number of positive factors and all other integers have an even number. [Editor's note: All the positive factors of an integer $n$ which is not a square can be paired in the following way: If $d$ is a factor of $n$ then $d$ is paired with $n/d$. This is the reason non-squares have an even number of positive factors. If $n = m^2$ then for $d = m$ we have that $d = n/d$, thus all factors except $m$ can be paired and thus there is an odd number of factors.]

As the square root of 1,000 is between 31 and 32, the 31 lights represented by perfect squares will be on at the end of the exercise.