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Problem of the Week

Proposed by Bernardo Ábrego and Silvia Fernández. **September 20-27**

Today, September 20, 2004, is my birthday and I noticed a curious fact: My father's age (in whole years) is exactly twice mine (also in whole years). I wonder how many days, including today, can this happen. In other words, how many days can my father's age be exactly twice my age? Since you do not know my age or my father's, I will at least tell you that my father's birthday is not February 29^{th}.
Make sure you explain why your answer is correct.

Solution by Boian Djonov. Let the age of the son be x years and the age of the father y years (on that particular September, 20). On the son's birthday (September, 20), $y = 2x$ (the father's age is twice his son's).This relation of the ages will last till the father's birthday when his age will be incremented by one and won't be anymore twice his son's age (if $y = 2x$, it is not possible that $y + 1 = 2x$ as well). So, the father's age will be exactly twice his son's age during the period between the son's birthday and the father's birthday. Let's call this period Period 1.

Next, there are two times in the year when the relation between the years of the son and the father change. Namely, these are the son's and the father's birthdays. Let's consider first the son' birthday (when the age of the father is twice the age of the son). The question is whether it is possible some k years later on that date the father's age to be again twice his son's age. Son's age after k years $= x + k$; father's age after k years $= y + k$; So, we set the equation $y + k = 2(x + k)$. But since $y = 2x$, the equation becomes $2x + k = 2(x + k)$, so $k = 0$. So, never on the son's birthday will the age of the father be again twice the age of the son.

Let's consider next the father's birthday. On that birthday (following immediately the birthday of the son when $y = 2x$), the age of the father would be $y + 1$, while the age of the son would still be x. So, the question is whether it is possible some m years later on that date the age of the father to be a twice the age of the son. Son's age after k years $= x + m$; father's age after k years $= y + 1 + m$; So, we set the equation $y + 1 + m = 2(x + m)$. Since $y = 2x$, the equation becomes $2x + 1 + m = 2(x + m)$. So, $2x + 1 + m = 2x + 2m$ and $m = 1$. In other words, one year after the birthday of the father (which follows immediately the birthday of the son when $y = 2x$) the age of the father would be again twice the age of the son. This relationship will last till the son's birthday when his age will be incremented, but not that of the father. Let's call this period between the father's birthday and the son's birthday Period 2.

So, in total, the father's age would be twice that of his son during Period 1 and Period 2. The total number of days in Period 1 + Period 2 (from the son's birthday till the father's birthday + from the father's birthday till the son's birthday) would be 365 (one year).