Problem of the Week.   September 13-20

Proposed by Bernardo Ábrego and Silvia Fernández.

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30 ants are located at random on top of a narrow wooden stick measuring 12 inches long. Each ant starts walking, either left-bound or right-bound, at a speed of 2 inches per second (each ant's direction is chosen at random). Whenever two ants collide they immediately change directions while maintaining their speed. When an ant reaches either end of the stick, it falls off it. If all 30 ants start walking at the same time, prove that none of the ants remains on the stick after 6 seconds.

Solution by Robert Reiner. Because all of the ants are travelling at the same speed, the situation is identical with one where, when one ant hits another, instead of changing directions with an ant, the two ants pass each other by and continue on their way at their normal pace. Thus since every ant would ideally travel 12 inches in 6 seconds, it is certainly true that at the end of 12 seconds, all the ants would have traveled over one of the ends.

There is in fact no difference in my situation and the situation of the problem if we can not distinguish between the ants at all. (And even if we can distinguish between them, that does not mean that the situation will be any different.)