Problem of the Week. **August 30-September 6**

Proposed by Bernardo Ábrego and Silvia Fernández.

Find all integers n, such that

$$\frac{9n^2+1955}{3n-7}$$

is also an integer. Justify why your solution includes ALL possible values of n.

Solution by Chris Dungan.

Let $x_n = (9n^2 + 1955) / (3n - 7)$
$= [(9n^2 - 49) + 2004] / (3n - 7)$
$= [(3n - 7)(3n + 7) / (3n - 7)] + [2004 / (3n - 7)]$
$= (3n + 7) + [2004 / (3n - 7)]$

Since n is an integer, $(3n + 7)$ must be an integer.

For x_n to be an integer, since $(3n + 7)$ is an integer,
$\{[2004 / (3n - 7)] = [(167)(3)(2)(2)(1) / (3n - 7)]\}$ must be an integer.

Therefore, for x_n to be integer, $(3n - 7)$ must be evenly divisible by a factor of 2004. (Funny someone should think of this problem this year).

The entire set of numbers which 2004 is evenly divisible consists of 2004, 1002, 668, 501, 334, 167, 12, 6, 4, 3, 2, 1, -1, -2, -3, -4, -6,-12, -167, -334, -501, -668, -1002, and -2004.

As the denominator $(3n - 7)$ must be an integer which is 7 less than a number evenly divisible by 3, the above set must be reduced to the following subset: 668, 167, 2, -1, -4 and -334.

For $(3n - 7)$ to equal these six factors, n must respectively equal 225, 58, 3, 2, 1, and -109.