California State University Northridge

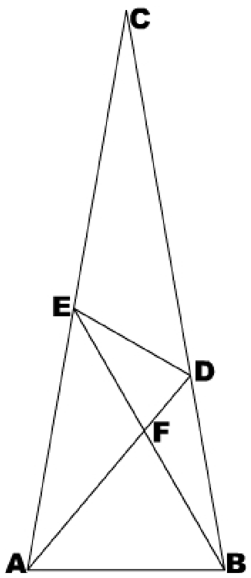
Department of Mathematics

Problem of the Week

Proposed by Miroslav Tanushev. **November 1-8**

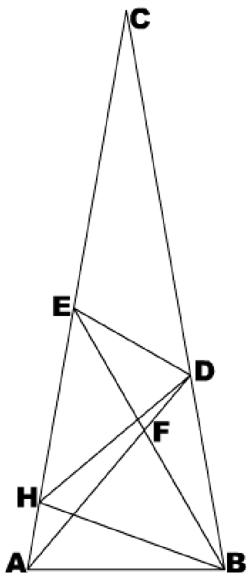
Let ABC be an isosceles triangle with $BC = CA$ and $\angle BCA = 20°$. Points D and E are on the sides BC and CA, respectively, and they satisfy that $\angle DAB = 50°$ and $\angle ABE = 60°$. Find, with proof, the exact value of the angle $\angle DEB$.

Solution by: Jeffrey Liu



We know $\angle BCA = 20°$ and $AC = BC$, $\angle DAB = 50°$ and $\angle ABE = 60°$. From above we know $\angle DAE = 30°$ $((180° - 20°)/2 - 50° = 30°)$, $\angle AFB = 70°$ $(180° - 50° - 60° = 70°) = \angle EFD$, $\angle EBD = 20°$ $((180° - 20°)/2 - 60° = 20°)$, $\angle ADB = 50°$ $(70° - 20° = 50°)$, and $\angle AEB = 40°$ $(180° - 60° - 50° - 30° = 40°)$.

From $\angle BDA = 50° = \angle BAD$, we know $BD = AB$.



Find a point H, make $\angle HBA = 20°$, draw line HB and HD. $\angle BHA = \angle BAH = 80°$ $(180° - 20° - 80° = 80°)$, so that $AB = BH$.

$\angle ABE = 40°$ $(60° - 20° = 40°)$, $\angle DBH = 60°$ $(40° + 20° = 60°)$. Triangle DHB is an equilateral triangle, $DH=HB=BD=BA$. And $\angle HEB = 40° = \angle HBE$, so $DH=HB=BD=BA=EH$

We know the $EH=DH=BH$, that mean point E, D and B are on the same circle, which point H is the center. We can say $\angle DHB = 2\angle DEB$, that mean $\angle DEB = 60°/2 = 30°$.

$\angle DEB = 30°$.