

Problem of the Week.

September 13-20

Proposed by Bernardo Ábrego and Silvia Fernández.

30 ants are located at random on top of a narrow wooden stick measuring 12 inches long. Each ant starts walking, either left-bound or right-bound, at a speed of 2 inches per second (each ant's direction is chosen at random). Whenever two ants collide they immediately change directions while maintaining their speed. When an ant reaches either end of the stick, it falls off it. If all 30 ants start walking at the same time, prove that none of the ants remains on the stick after 6 seconds.

Solution by Chris Dungan. Regardless of the number of ants on the stick, the fact that each changes direction upon colliding with one coming from the opposite direction has the same net effect as if each of such a pair continued in the same direction (only the individual ants have changed course, while the speed, position and direction are otherwise unaffected). So ants behaving in such a fashion would all have exited the 12" stick at either of the two ends by the time six seconds had elapsed because this is the longest they could have possibly walked in a straight line at 2" per second.