

Problem of the Week.

September 6-13

Proposed by Bernardo Ábrego and Silvia Fernández.

Suppose $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a matrix with a, b, c, d real numbers and $A^3 = \begin{pmatrix} 6 & 2 \\ 7 & 1 \end{pmatrix}$. Find a, b, c, d .

Note: A^3 represents the **matrix multiplication** of A with itself three times.

Additional questions for possible projects.

1. Which 2×2 matrices have n^{th} roots?, i.e., for which matrices A is there a matrix B so that $B^n = A$. This question can be investigated using rational, real, or complex entries.
2. In the alternative solution we proved there was a unique solution. If you think this is always the case, we encourage you to calculate A^3 with $A = \begin{pmatrix} 2 & 1 \\ -7 & -3 \end{pmatrix}$. This suggests the following problem: find all the n^{th} roots of the identity matrix. Again, this question can be done over the rational, the real, or the complex numbers.
3. Work problems 1 and 2 with larger matrices, 3×3 , 4×4 , or $m \times m$.