

Problem of the Week

Proposed by Bernardo Ábrego and Silvia Fernández.

October 11-18

Find explicit formulas for all functions f , from the positive integers to the real numbers, such that

$$f(n) + f(m) = f(n)f(m) + f(n+m)$$

for all positive integers n and m .

Additional questions for possible projects.

1. Find now all functions from the rational numbers to the real numbers satisfying the relation above. Is it the same answer?
2. Find all functions from the real numbers to the real numbers satisfying the relation above. You may want to add more hypothesis, like continuity or monotonicity.
3. What other doubly-recursive formulas can be used to get explicit formulas?