

Solutions to Exercise Four – Fins

1. A 25-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–6. Select a heat sink that will allow the case temperature of the transistor not to exceed 55°C in the ambient air at 18°C.

Using the thermal circuit Ohm's law analogy as an inequality – we want to have a resistance that just matches the Ohm's law value or a smaller one – gives the following result.

$$R \leq \frac{\Delta T}{\dot{Q}} \leq \frac{55^\circ C - 18^\circ C}{25 W} = \frac{1.48^\circ C}{W}$$

Looking at Table 3-6 we see that the following heat sinks have the required resistance: HS 5030 (either horizontal or vertical) HS 6071 (vertical) and HS 6115 (either horizontal or vertical). Final selection would be based on other factors on cost, space requirements, and integration with other design components

2. Consider a stainless steel spoon ($k = 8.7 \text{ Btu/h}\cdot\text{ft}\cdot^\circ\text{F}$) partially immersed in boiling water at 200°F in a kitchen at 75°F. The handle of the spoon has a cross section of 0.08 in by 0.5 in, and extends 7 in into the air from the free surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is 3 Btu/h·ft²·°F, determine the temperature difference across the exposed surface of the spoon handle. State your assumptions.



We assume that the spoon is a fin with a constant cross section of $(0.08 \text{ in})(0.5 \text{ in}) = 0.04 \text{ in}^2 = 0.0002778 \text{ ft}^2$ and a base temperature of 200°F. We assume negligible heat transfer from the contact between the spoon and the edge of the pot. The perimeter of the spoon is $2(0.08 \text{ in} + 0.5 \text{ in}) = 1.16 \text{ in} = 0.09667 \text{ ft}$. If we use the equation for a finite length fin with an end correction for convection, we have $L_c = L + A_c/p = 7 \text{ in} + (0.04 \text{ in}^2)/(1.16 \text{ in}) = 7.034 \text{ in}$. The equation for the temperature profile along a fin is given below.

$$\theta = \theta_b \frac{\cosh m(L-x)}{\cosh mL} \Rightarrow T - T_\infty = (T_b - T_\infty) \frac{\cosh m(L_c - x)}{\cosh mL_c} \quad \text{with} \quad m = \sqrt{\frac{hp}{kA}}$$

The parameter m can be computed from the data given in the problem and the values found above for perimeter and cross sectional area.

$$m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{3 \text{ Btu}}{\text{h}\cdot\text{ft}^2\cdot^\circ\text{F}} \left(\frac{\text{h}\cdot\text{ft}\cdot^\circ\text{F}}{8.7 \text{ Btu}} \right) \frac{0.09667 \text{ ft}}{0.0002778 \text{ ft}^2}} = \frac{10.954}{\text{ft}}$$

The temperature at the end of the spoon ($x = L_c$) is found as follows.

$$T_{x=L} = T_{\infty} + (T_b - T_{\infty}) \frac{\cosh m(L_c - L_c)}{\cosh mL_c} = 75^{\circ}F + (200^{\circ}F - 75^{\circ}F) \frac{1}{\cosh\left(\frac{10.954}{ft} (7.034 in) \frac{ft}{12 in}\right)} = 75.4^{\circ}F$$

So the temperature difference across the spoon, $T_b - T_{x=L} = 200^{\circ}F - 75.4^{\circ}F = \boxed{124.6^{\circ}F}$.

3. A hot surface at $100^{\circ}C$ is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins ($k = 237 \text{ W/m}\cdot^{\circ}C$) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is $30^{\circ}C$, and the heat transfer coefficient on the surfaces is $35 \text{ W/m}^2\cdot^{\circ}C$. Determine the rate of heat transfer from the surface for a 1-m by 1-m section of the plate. Also determine the overall effectiveness of the fins.

Here we have to consider the analysis for each fin then evaluate the overall surface consisting of fins and a remaining area without fins. For each fin the cross sectional area $A_c = \pi(0.0025 \text{ m})^2/4 = 4.909 \times 10^{-6} \text{ m}^2$ and the perimeter $p = \pi(0.0025 \text{ m}) = 0.007854 \text{ m}$. The corrected length, $L_c = L + A_c/p = 0.03 \text{ m} + (4.909 \times 10^{-6} \text{ m}^2) / (0.007854 \text{ m}) = 0.030625 \text{ m}$. With these data and the values for k and h given above we can compute the parameters m and mL_c .

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{35 \text{ W}}{\text{m}^2\cdot^{\circ}C} \left(\frac{\text{m}\cdot^{\circ}C}{237 \text{ W}}\right) \frac{0.007854 \text{ m}}{4.909 \times 10^{-6} \text{ m}^2}} = \frac{15.37}{\text{m}} \quad mL_c = \frac{15.37}{\text{m}} (0.030625 \text{ m}) = 0.4708$$

The heat transfer from one fin is given by the following equation.

$$\dot{Q}_{one \text{ fin}} = \sqrt{kA_c hp} (T_b - T_{\infty}) \tanh mL = \sqrt{\frac{35 \text{ W}}{\text{m}^2\cdot^{\circ}C} (4.909 \times 10^{-6} \text{ m}^2) \frac{237 \text{ W}}{\text{m}\cdot^{\circ}C} (0.007854 \text{ m})} (100^{\circ}C - 30^{\circ}C) \tanh(0.4708) = 0.5493 \text{ W}$$

The fins with a diameter of 0.0025 m, separated at a distance 0.06 m between centers, have a distance from the start of one fin to the start of the next fin of 0.06 m. Over a length of 1 m there is room for $1/0.06 = 166$ fins. Over the 1-m-square plate there will be a total of $166^2 = 27556$ fins. The plate area occupied by these fins is $27556(4.909 \times 10^{-6} \text{ m}^2) = 0.1353 \text{ m}^2$. The remaining unfinned area = $1 \text{ m}^2 - 0.1353 \text{ m}^2 = 0.8647 \text{ m}^2$ will have a convective heat transfer with the same heat transfer coefficient and temperature difference. Thus, the total heat transfer is found as follows.

$$\begin{aligned} \dot{Q} &= \dot{Q}_{finned} + \dot{Q}_{unfinned} = N_{fin} \dot{Q}_{one \text{ fin}} + hA_{unfinned} (T_b - T_{\infty}) \\ &= (27556)(0.5493 \text{ W}) + \frac{35 \text{ W}}{\text{m}^2\cdot^{\circ}C} (0.8647 \text{ m}^2) (100^{\circ}C - 30^{\circ}C) = 172555 \text{ W} \end{aligned}$$

$$\boxed{\dot{Q} = 17.26 \text{ kW}}$$

Without fins the heat transfer would be with the same convection coefficient and temperature difference over an area of 1 m^2 .

$$\dot{Q}_{no \text{ fins}} = hA_{no \text{ fins}} (T_b - T_{\infty}) = \frac{35 \text{ W}}{\text{m}^2\cdot^{\circ}C} (1 \text{ m}^2) (100^{\circ}C - 30^{\circ}C) = 2450 \text{ W}$$

The overall effectiveness is the ratio $(17.26 \text{ kW}) / (2.45 \text{ kW}) = \boxed{7.04}$.