

Solutions to In-class Exercise One

1. The inner and outer surfaces of a 0.5-cm thick 2-m by 2-m window glass in winter are 10°C and 3°C, respectively. If the thermal conductivity of the glass is 0.78 W/m·K, determine the amount of heat loss through the glass over a period of 5 h. What would your answer be if the glass were 1 cm thick? (Problem 1.56 from text.)

This is a basic conduction problem where $\dot{Q} = kA\Delta T/L$. The heat loss from the inside to the outside is found by taking $\Delta T = 10^\circ\text{C} - 3^\circ\text{C} = 7^\circ\text{C} = 7\text{ K}$, the inside to outside temperature difference. The area, A, is $(2\text{ m})(2\text{ m}) = 4\text{ m}^2$, and we are given the thermal conductivity, $k = 0.78\text{ W/m}\cdot\text{K}$. We are asked to find the heat for two values of L: 0.5 cm = 0.005 m and 1 cm = 0.01 m. The total heat transferred over five hours is found by assuming that the temperatures are constant over that period so that $Q = \dot{Q}\Delta t$, where $\Delta t = 5\text{ hr} = 18,000\text{ s}$. Combining these equations and applying the data gives the following results for L = 0.5 cm and L = 1 cm.

$$Q = \dot{Q}\Delta t = \frac{kA\Delta T}{L}\Delta t = \frac{0.78\text{ W}}{\text{m}\cdot\text{K}} \frac{(4\text{ m}^2)(7\text{ K})}{0.005\text{ m}} (18000\text{ s}) \frac{1\text{ J}}{\text{W}\cdot\text{s}} \frac{\text{MJ}}{10^6\text{ J}} = 78.6\text{ MJ}$$

$$Q = \dot{Q}\Delta t = \frac{kA\Delta T}{L}\Delta t = \frac{0.78\text{ W}}{\text{m}\cdot\text{K}} \frac{(4\text{ m}^2)(7\text{ K})}{0.01\text{ m}} (18000\text{ s}) \frac{1\text{ J}}{\text{W}\cdot\text{s}} \frac{\text{MJ}}{10^6\text{ J}} = 39.3\text{ MJ}$$

The second answer can also be found by dividing the first answer by 2 to account for the doubling of the thickness of the glass.

2. Hot air at 80°C is blown over a 2-m by 4-m flat surface at 30°C. If the average convection heat transfer coefficient is 55 W/m²·°C, determine the rate of heat transfer from the air to the plate, in kW. (Problem 1.67 from text.)

Here we apply the basic equation for convection, $\dot{Q} = hA\Delta T$. We are asked to find the heat transfer from the air to the plate so we take the temperature difference as the air temperature minus the plate temperature = 80°C – 30°C = 50°C. The area is $(2\text{ m})(4\text{ m}) = 8\text{ m}^2$, and the heat transfer coefficient, $h = 55\text{ W/m}^2\cdot^\circ\text{C}$. Substituting these data into the convective heat transfer equation gives the desired heat transfer:

$$\dot{Q} = hA\Delta T = \frac{55\text{ W}}{\text{m}^2\cdot^\circ\text{C}} (8\text{ m}^2)(50^\circ\text{C}) \frac{\text{kW}}{10^3\text{ W}} = 22\text{ kW}$$

3. Consider a person whose exposed surface area is 1.7 m^2 , emissivity is 0.5, and surface temperature is 32°C . Determine the rate of heat loss from that person by radiation in a large room having walls at a temperature of (a) 300 K and (b) 280 K. (Problem 1.84 from text.)

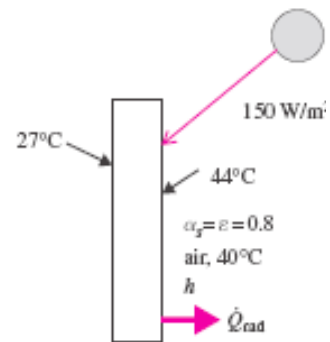
Solution done for original problem in text with $A = 1.7 \text{ m}^2$ instead of 0.7 m^2 on exercise sheet.

For this radiation problem we can assume that the person is small compared to the size of the room and use the formula for the radiative heat transfer from a small object (1) to a large enclosure (2), $\dot{Q} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)$. We are given the emissivity, $\varepsilon_1 = 0.5$, the surface area, $A_1 = 1.7 \text{ m}^2$, and $T_1 = 32^\circ\text{C} = 305.15 \text{ K}$. We are asked to use two values of T_2 : 300 K and 280 K. The heat transfer from these two temperatures are.

$$\dot{Q} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4) = (0.5)(1.7 \text{ m}^2) \frac{5.670 \times 10^{-8} \text{ W}}{\text{m}^2 \cdot \text{K}^4} \left[(305.15 \text{ K})^4 - (300 \text{ K})^4 \right] = 26.7 \text{ W}$$

$$\dot{Q} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4) = (0.5)(1.7 \text{ m}^2) \frac{5.670 \times 10^{-8} \text{ W}}{\text{m}^2 \cdot \text{K}^4} \left[(305.15 \text{ K})^4 - (280 \text{ K})^4 \right] = 121 \text{ W}$$

4. The inner and outer surfaces of a 25-cm-thick wall in summer are at 27°C and 44°C , respectively. (See diagram at right.) The outer surface of the wall exchanges heat by radiation with surrounding surfaces at 40°C , and convection with ambient air also at 40°C with a convection heat transfer coefficient of $8 \text{ W/m}^2 \cdot ^\circ\text{C}$. Solar radiation is incident on the surface at a rate of 150 W/m^2 . If both the emissivity and the solar absorptivity of the outer surface are 0.8, determine the effective thermal conductivity of the wall. (Problem 1.97 from text.)



Here there is conduction heat transfer through the wall that equals the heat transfer from the outside of the wall. We thus have the balance equation that $\dot{q}_{cond} = \dot{q}_{rad} + \dot{q}_{conv}$. Here, we choose to use heat fluxes, instead of heat flows because we are not given the area. We have to be careful to check the sign of each component of the conductive and radiative heat transfer so that we get the correct net heat transfer into the wall.

Once we find the conduction heat flux from this sum, we can find the thermal conductivity by solving the usual equation for conduction heat transfer, $\dot{q} = k\Delta T/L$, to give the thermal conductivity, $k = L\dot{q}/\Delta T$. We assume that the heat transfer is going from the outer wall at 44°C to the inner wall at 27°C so $\Delta T = 17^\circ\text{C}$ and the wall thickness, $L = 25 \text{ cm} = 0.25 \text{ m}$.

On the outside of the wall we have an incoming solar radiation of 150 W/m^2 . Since the solar absorptivity $\alpha = 0.8$, the heat transfer that is actually absorbed by the wall is $(0.8)(150 \text{ W/m}^2) = 120 \text{ W/m}^2$. In addition to this solar irradiation, there is a radiative heat exchange with other surfaces at 40°C . Assuming that the other surfaces are larger than the wall, we can use the equation for radiative transfer from a small body in a large enclosure to find the radiative heat exchange per unit area.

$$\dot{q}_{rad,x} = \frac{\dot{Q}}{A_1} = \varepsilon_1 \sigma (T_1^4 - T_1^4) = (0.8) \frac{5.670 \times 10^{-8} \text{ W}}{\text{m}^2 \cdot \text{K}^4} \left[(317.15 \text{ K})^4 - (313.15 \text{ K})^4 \right] = 22.72 \text{ W}$$

This heat transfer is leaving the wall, not going into the wall because the wall temperature is greater than the surrounding temperatures. Thus the total radiation flux into the wall = $120 \text{ W/m}^2 - 22.72 \text{ W/m}^2 = 97.28 \text{ W/m}^2$

The convective heat transfer into the wall is found by the equation for convection, $\dot{Q} = hA\Delta T$.

Dividing by A gives the heat flux and to get the heat flux into the wall we take the temperature difference as $T_{\text{air}} - T_{\text{outer,wall}} = 40^\circ\text{C} - 44^\circ\text{C} = -4^\circ\text{C}$. With the given heat transfer coefficient of $8 \text{ W/m}^2 \cdot ^\circ\text{C}$, we find the heat flux into the wall as

$$\dot{q}_{conv} = \frac{\dot{Q}_{conv}}{A} = h\Delta T = \frac{8 \text{ W}}{\text{m}^2 \cdot ^\circ\text{C}} (-4^\circ\text{C}) = -\frac{32 \text{ W}}{\text{m}^2}$$

So the total heat flux into the wall at the outside, which equals the conduction heat flux through the wall is found as follows.

$$\dot{q}_{cond} = \dot{q}_{rad} + \dot{q}_{conv} = \frac{97.28 \text{ W}}{\text{m}^2} + \frac{-32 \text{ W}}{\text{m}^2} = \frac{65.28 \text{ W}}{\text{m}^2}$$

We can now solve the conduction heat transfer equation for the thermal conductivity and apply the given data ($L = 0.25 \text{ m}$ and $\Delta T = 44^\circ\text{C} = 27^\circ\text{C} = 17^\circ\text{C} = 17 \text{ K}$) to find the thermal conductivity.

$$\dot{q} = \frac{k\Delta T}{L} \Rightarrow k = \frac{L\dot{q}}{\Delta T} = \frac{(0.25 \text{ m}) \left(\frac{65.28 \text{ W}}{\text{m}^2} \right)}{17 \text{ K}} = \frac{0.96 \text{ W}}{\text{m} \cdot \text{K}}$$