Solution to Fourth Quiz – Fins

You are asked to evaluate two possible designs for fin cooling. One design uses a single rectangular fin that is 4.1 cm wide, 0.5 cm thick and has a length of 3 cm. An alternative design, which requires the same space, but less material, consists of four cylindrical fins with a diameter of 0.5 cm and a separation distance between cylinders of 0.7 cm. Both designs use aluminum (k = 237 W/m·°C), have a heat transfer coefficient of 35 W/m²·°C, a base temperature of 100°C, and an ambient temperature of 30°C. Which design has the higher heat transfer?

For the single rectangular fin with w = 0.041 m, and t = 0.005 m, the perimeter is 2(w+t) = 2(0.041 m + 0.005 m) = 0.092 m. The cross-sectional area, A_c = tw = (0.041 m)(0.005 m) = 0.000205 m^2. The corrected length $L_c = L + A_c/p$ = 0.03 m + (0.000205 m^2)/(0.092 m) = 0.03223 m. With these data and the values of k and h given in the problem statement, we can compute the values of m and mL_c for the rectangular fin.

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{35\,W}{m^2\cdot{}^oC}\left(\frac{m\cdot{}^oC}{237\,W}\right)\frac{0.092\,m}{0.000205\,m^2}} = \frac{8.141}{m} \qquad mL_c = \frac{8.141}{m}(0.03223\,m) = 0.2624$$

The heat transfer from this rectangular fin can now be found.

$$\dot{Q}_{rect\ fin} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL =$$
$$\sqrt{\frac{35\,W}{m^2\cdot{}^oC}(0.000205\,m^2)\frac{237\,W}{m\cdot{}^oC}(0.0092\,m)}\left(100^oC - 30^oC\right)\tanh(0.2624) = 7.1023\,W$$

Each of the four cylindrical fins has a cross sectional area $A_c = \pi(0.005\text{ m})^2/4 = 19.635\text{x}10^{-6}\text{ m}^2$ and the perimeter p = π(0.005 m) = 0.01571 m. The corrected length, $L_c = L + A_c/p$ = 0.03 m + (19.635x10^{-6} m^2) / (0.01571 m) = 0.03125 m. With these data and the values for k and h given above we can compute the parameters m and mL_c for each cylindrical fin.

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{35\,W}{m^2\cdot{}^oC}\left(\frac{m\cdot{}^oC}{237\,W}\right)\frac{0.01571\,m}{10.635x10^{-6}\,m^2}} = \frac{10.87}{m} \qquad mL_c = \frac{10.872}{m}(0.03125\,m) = 0.3397$$

The heat transfer from one cylindrical fin is given by the following equation.

$$\dot{Q}_{one\ cyl} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL =$$
$$\sqrt{\frac{35\,W}{m^2\cdot{}^oC}(19.635x10^{-6}\,m^2)\frac{237\,W}{m\cdot{}^oC}(0.01571\,m)}\left(100^oC - 30^oC\right)\tanh(0.3397) = 1.158\,W$$

Compared to base area of the rectangular fins, the cylindrical fins will have an unfinned area of tw – 4[$\pi D^2/4$] = 0.000205 m^2 – 4(19.635x10^{-6} m^2) = 0.000126 m^2 from which there will be convection, Thus the total convection from the cylindrical fins will be the sum of the heat transfer from the four fins plus the heat transfer from the unfinned base:

$$\dot{Q} = \dot{Q}_{finned} + \dot{Q}_{unfinned} = N_{fin}\dot{Q}_{one\ fin} + hA_{unfinned}(T_b - T_\infty)$$
$$= (4)(1.158\ W) + \frac{35\ W}{m^2 \cdot {}^oC}\left(0.000126\ m^2\right)\left(100^oC - 30^oC\right) = 4.942\ W$$

The single rectangular fin has the greater heat transfer.