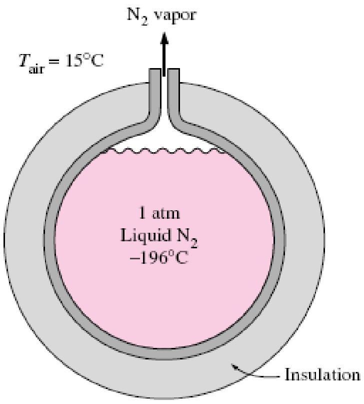
Solution to Quiz Three – Steady Heat Transfer



Liquid nitrogen in a spherical steel tank (k = 60 W/m·K) with an inside wall thickness of 2 cm and an outer diameter of 3 m has an opening to the atmosphere. Because of this opening, the temperature of the liquid nitrogen in the tank remains constant at –196°C, the normal boiling point of nitrogen. Heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg The tank is insulated with 5-cm-thick fiberglass insulation (*k* = 0.035 W/m·°C) and placed in an ambient temperature of 15°C. Radiation in the tank is negligible and the inside convection coefficient is 25,000 W/m^2·°C. The tank exterior has a combined convection and radiation heat transfer coefficient of 35 W/m^2·°C. Compute the evaporation rate (a) considering all resistances and (b) ignoring the resistance due to the interior convection and the conduction through the steel. What is the error due to ignoring the resistances in the tank interior and the steel?

The equivalent circuit for considering all resistances is shown at the left. It has resistances for (1) convection inside the tank, (2) condution through the steel, (3) conduction through the insulation, and (4) conduction plus radiation to the ambient air.

Considering all four resistances, the heat transfer from the ambient air into the liquid nitrogen is given by the following equation, where r_i and r_o are the inner and outer radii of the steel container and r_{ins} is the outer radius of the insulation. (The inner radius of the insulation is r_o.)

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_i + R_1 + R_2 + R_o} = \frac{T_{\infty 2} - T_{\infty 1}}{\frac{1}{h_i 4\pi r_i^2} + \frac{r_o - r_i}{4\pi k r_i r_o} + \frac{r_{ins} - r_o}{4\pi k r_o r_{ins}} + \frac{1}{h_{ins} 4\pi r_{ins}^2}}$$

Substituting the data given in the problem gives

$$\dot{Q} = \frac{4\pi\left[15^oC - \left(-196^oC\right)\right]}{\frac{1}{\frac{25000\,W}{m^2\cdot{}^oC}(1.48\,m)^2} + \frac{1.5\,m - 1.48\,m}{\frac{60\,W}{m\cdot{}^oC}(1.48\,m)(1.5\,m)} + \frac{1.55\,m - 1.5\,m}{\frac{0.035\,W}{m\cdot{}^oC}(1.55\,m)(1.5\,m)} + \frac{1}{\frac{35\,W}{m^2\cdot{}^oC}(1.55\,m)^2}}$$

$$\dot{Q} = \frac{4\pi[211]}{0.000183 + 0.000150 + 0.61449 + 0.011892} W = 4231\,W$$

The evaporation rate is the heat transfer rate divided by the latent heat.

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$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4231\,W}{\frac{198\,kJ}{kg}} \frac{kJ}{1000\,W \cdot s} = \frac{0.02137\,kg}{s}$$

If we ignore the first two resistances, the heat transfer and the evaporate rate become

$$\dot{Q} = \frac{4\pi[211]}{0.61449 + 0.011892} W = 4233\,W$$

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4233\,W}{\frac{198\,kJ}{kg}} \frac{kJ}{1000\,W \cdot s} = \frac{0.02138\,kg}{s}$$

The difference between the evaporation rates is 1.1×10^{-5} kg/s, a ***relative error of 0.053%***.