May 2 Homework Solutions

12-14 A cordless telephone is designed to operate at a frequency of 8.5x10⁸ Hz. Determine the wavelength of these telephone waves.

We can use the relationship between wavelength, frequency, and the speed of light, c = 2.998x10⁸ m/s.

$$\lambda = \frac{c}{\nu} = \frac{\frac{2.998x10^8 m}{s}}{\left(8.5x10^8 Hz\right)\frac{1}{Hz \cdot s}} = \boxed{0.353\ m}$$

12-20 Consider a cube, 20 cm on a side, that is suspended in air and emitting radiation, that closely approximates a black body, at T = 750 K. Find (a) the rate at which the cube emits energy in W, (b) the spectral black-body emissive power at a wavelength of 4 μm.

We can use the Stefan-Boltzmann formula for the emissive power of a black-body: $E = \sigma AT^4$, where the area of the cube is six times the area of each side = $6(0.2\ m)^2 = 0.24\ m^2$. The radiation then is

$$E = \sigma AT^4 = \frac{5.670x10^{-8} W}{m^2 \cdot K^4}\left(0.24\ m^2\right)\left(750\ K\right)^4 = \boxed{4306\ W}$$

The spectral radiation is found from the following formula

$$E_{b,\lambda} = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)} = \frac{\frac{3.74177\ W \cdot \mu m^4}{m^2}}{\left(4\mu m\right)^5\left(e^{(1.43878 \mu m \cdot K)/[(4\mu m)(750 K)]} - 1\right)} = \boxed{3045\ W/m^2 \cdot \mu m}$$

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12-48 The emissivity of a surface coated with aluminum oxide can be approximated to be 0.15 for radiation at wavelengths less than 5 μm and 0.9 for radiation greater than 5 μm. Determine the average emissivity of this surface at (a) 5800 K and (b) 300 K. What can you say about the absorptivity of this surface for radiation coming from sources at 5800 K and 300 K?

For both temperatures the average emissivity, ε, is given by the general equation (which is the first integral on the right) that is subsequently applied to the profile in this problem. In this equation ε_1 = 0.15 and ε_2 = 0.9.

$$\varepsilon = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda E_{b,\lambda} d\lambda = \frac{1}{\sigma T^4}\int_0^{5\,\mu m} \varepsilon_1 E_{b,\lambda} d\lambda + \frac{1}{\sigma T^4}\int_{5\,\mu m}^\infty \varepsilon_2 E_{b,\lambda} d\lambda =$$
$$\varepsilon_1 f_\lambda[(5\ \mu m)T] + \varepsilon_2\{1 - f_\lambda[(5\ \mu m)T]\} = \varepsilon_2 + (\varepsilon_1 - \varepsilon_2) f_\lambda[(5\ \mu m)T]$$

For T = 5800 K, (5 μm)T = (5 μm)(5800 K) = 29,000 μm·K; at this value of λT, f_λ = 0.994715 from interpolation in Table 12-2 on page 672 of the text. Thus we have the following result for T = 5800 K.

$$\varepsilon_{5800\,K} = \varepsilon_2 + 0.994715\,(\varepsilon_1 - \varepsilon_2) = 0.9 + 0.994715\,(0.15 - 0.9) = \boxed{0.15}$$

For T = 300 K, (5 μm)T = (5 μm)(300 K) = 1500 μm·K; at this value of λT, f_λ = 0.013754 from interpolation in Table 12-2. Thus we have the following result for T = 300 K.

$$\varepsilon_{300K} = \varepsilon_2 + 0.013574(\varepsilon_1 - \varepsilon_2) = 0.9 + 0.013574(0.15 - 0.9) = \boxed{0.89}$$

From Kirchoff's law we know that the spectral absorptivity equals the spectral emissivity (assuming a diffuse surface): $\alpha_\lambda = \varepsilon_\lambda$l. Since we have integrated over the entire black-body radiation spectrum here we know that the total absorptivity will equal the total emissivity. Thus **_α = 0.15 for radiation at 5800 K and α = 0.89 for radiation at 300 K_**.

12-50E A 5-in diameter spherical ball is known to emit radiation at a rate of 550 But/h when its surface temperature is 950 R. Determine the average emissivity of the ball at this temperature.

We can use the formula for the emissive power of a body with a given emissivity ε: E = $\varepsilon\sigma AT^4$, where the surface area of the ball equals πD^2. Solving for the emissivity and substituting values gives the result as follows.

$$\varepsilon = \frac{E}{\sigma AT^4} = \frac{E}{\sigma\pi D^2 T^4} = \frac{\frac{550\ Btu}{h}}{\frac{0.1714x10^8\ Btu}{h\cdot ft^2\cdot R^4}\pi\left[(5\ in)\frac{1\ ft}{12\ in}\right]^2(950\ R)^4} = \boxed{0.722}$$

12-61E Solar radiation is incident on the outer surface of a spaceship at a rate of 400 Btu/h·ft². The surface has an absorptivity of α_s = 0.10 for solar radiation and an emissivity of ε = 0.6 at room temperature. The outer surface radiates heat into space at 0 R. If there is no net heat transfer into the spaceship, determine the equilibrium temperature of the surface.

The equilibrium temperature of the spaceship occurs when the incoming solar radiation absorbed by the surface equals the radiation emitted into space. (There is no convection in the vacuum of outer space.). Writing this radiation balance gives.

$$\alpha_s G_{solar} = \varepsilon\sigma\left(T_{ship}^4 - T_{space}^4\right) \quad \Rightarrow \quad T_{ship} = \sqrt[4]{T_{space}^4 + \frac{\alpha_s G_{solar}}{\varepsilon\sigma}}$$

$$= \sqrt[4]{(0\ R)^4 + \frac{(0.10)\frac{400\ Btu}{h\cdot ft^2}}{(0.6)\frac{0.1714x10^8\ Btu}{h\cdot ft^2\cdot R^4}}} = \boxed{444\ R}$$

12-63 The absorber surface of a solar collector is made of aluminum coated with black chrome (α_s = 0.87 and ε = 0.09). Solar radiation is incident on the surface at a rate of 600 W/m². The air and effective sky temperatures are 25°C and 15°C, respectively, and the convection heat transfer coefficient is 10 W/m²·°C. For an absorber surface temperature of 70°C, determine the net rate of solar energy delivered by the absorber plate to the water circulating behind it.

The net heat flux from the absorber plate into the circulating water is the difference between the solar energy absorbed by the collector and the energy lost by convection and radiation.

$$\dot{q}_{collector} = \dot{q}_{solar} - \dot{q}_{lost} = \alpha_s G_{solar} - h(T_s - T_\infty) - \varepsilon\sigma\left(T_s^4 - T_{sky}^4\right)$$

All the data on the right-hand side of this equation are known, so we can solve for the heat flux to the collector. Note that we can use temperatures in degrees Celsius for the convection term, but we have to convert the temperatures to kelvins for the radiation term.

$$\dot{q}_{collector} = (0.87)\frac{600\,W}{m^2} - \frac{10\,W}{m^2\cdot{}^oC}(70^oC - 25^oC) - (0.09)\frac{5.670x10^{-8}\ W}{m^2\cdot K^4}\left[(343.15\ K)^4 - (288.15\ K)^4\right]$$

$$\dot{q}_{collector} = \boxed{36.5\ W/m^2}$$