California State University Northridge

College of Engineering and Computer Science
Mechanical Engineering Department

Mechanical Engineering 375
Heat Transfer

Spring 2007 Number 17629 Instructor: Larry Caretto

March 7 Homework Solutions

4-14 The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2-mm-diameter sphere. The properties of the junction are k = 35 W/m·°C, ρ = 8500 kg/m^3, and c_p = 320 J/kg·°C; the heat transfer coefficient between the junction and the gas is h = 90 W/m^2·°C. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.

First we compute the characteristic length and the Biot number to see if the lumped parameter analysis is applicable.

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{6}D^3}{\pi D^2} = \frac{D}{6} = \frac{1.2\ mm}{6} = \frac{0.0012\ m}{6} = 0.0002\ m$$

$$Bi = \frac{hL_c}{k} = \frac{90\,W}{m^2\cdot{}^oC}(0.0002\ m)\left(\frac{m\cdot{}^oC}{35\,W}\right) = 0.0005 < 0.1$$

Since the Biot number is less than 0.1, we can use the lumped parameter analysis. In such an analysis, the time to reach a certain temperature is given by the following equation.

$$t = -\frac{1}{b}\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) \qquad b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

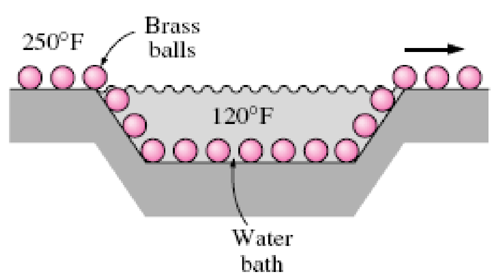
From the data in the problem we can compute the parameter, b, and then compute the time for the ratio $(T - T_\infty)/(T_i - T_\infty)$ to reach the desired value.

$$b = \frac{h}{\rho c_p L_c} = \frac{90\,W}{m^2\cdot{}^oC}\frac{m^3}{8500\ kg}\frac{kg\cdot{}^oC}{320\ J}\frac{1}{0.0002\ m}\frac{1\ J}{W\cdot s} = \frac{0.1654}{s}$$

The problem statement is interpreted to read that the measured temperature difference $T - T_\infty$ has eliminated 99% of the transient error in the initial temperature reading $T_i - T_\infty$; so the value of value of $(T - T_\infty)/(T_i - T_\infty)$ to be used in this equation is 0.01. Substituting this value and the value for b just found gives the following result for the time.

$$t = -\frac{1}{b}\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -\frac{s}{0.1654}\ln(0.01) = \boxed{27.8\ s}$$

4-15E In a manufacturing facility, 2-in-diameter brass balls (k = 64.1 Btu/h ft·°F, ρ = 532 lb$_m$/ft^3, and c_p = 0.092 Btu/lb$_m$·°F) initially at 250°F are quenched in a water bath at 120°F for a period of 2 min at a rate of 120 balls per minute. If the convection heat transfer coefficient is 42 Btu/h ft^2·°F, determine (a) the temperature of the



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balls after quenching and (b) the rate at which heat needs to be removed from the water in order to keep its temperature constant at 120°F.

First we compute the characteristic length and the Biot number to see if the lumped parameter analysis is applicable.

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{6}D^3}{\pi D^2} = \frac{D}{6} = \frac{2\ in}{6}\frac{ft}{12\ in} = 0.02778\ ft$$

$$Bi = \frac{hL_c}{k} = \frac{42\ Btu}{h\cdot ft^2\cdot {}^oF}(0.02278\ ft)\left(\frac{h\cdot ft\cdot {}^oF}{64.1\ Btu}\right) = 0.018 < 0.1$$

Since the Biot number is less than 0.1, we can use the lumped parameter analysis. The temperature after a time, t, is given by the following equation.

$$(T - T_\infty) = (T_i - T_\infty)e^{-bt} \qquad b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

From the data in the problem we can compute the parameter, b, and then compute the temperature after 2 minutes.

$$b = \frac{h}{\rho c_p L_c} = \frac{42\ Btu}{h\cdot ft^2\cdot {}^oF}\frac{ft^3}{532\ lb_m}\frac{lb_m\cdot {}^oF}{0.092\ Btu}\frac{1}{0.02778\ ft} = \frac{30.9}{h}$$

From the problem data we have T_∞ = 120°F and T_i = 250°F. The temperature after a quenching time of 2 minutes is found as follows.

$$T = T_\infty + (T_i - T_\infty)e^{-bt} = 120^o F + \left(250^o F - 120^o F\right)e^{-\frac{30.9}{h}(2\ \mathrm{min})\frac{1\ h}{60\ \mathrm{min}}} = \boxed{166^o F}$$

The heat transfer to each ball is the mass times the heat capacity times the difference between the initial and final temperature.

$$Q_{ball} = mc_p(T - T_i) = \rho V c_p(T - T_i) = \rho\frac{\pi D^3}{6}c_p(T - T_i) =$$

$$\frac{532\ lb_m}{ft^3}\frac{\pi}{6}\left[(2\ in)\frac{ft}{12\ in}\right]^3\frac{0.092\ Btu}{lb_m\cdot {}^oF}\left(250^o F - 166^o F\right) = 9.97\ Btu$$

For 120 balls per minute the total heat removal is (120/minute)(9.97 Btu) = **_1196 Btu/min_**.

4-19 A long copper rod of diameter 2.0 cm is initially at a uniform temperature of 100°C. It is now exposed to an air stream at 20°C with a heat transfer coefficient of 200 W/m²·K. How long would it take for the copper rod to cool to an average temperature of 25°C?

Before we can compute the characteristic length and the Biot number to see if the lumped parameter analysis is applicable, we must first find the properties of copper from Table A-2 in the text: k = 401 W/m·°C, ρ = 8933 kg/m³, and c_p = 385 J/kg·°C. (Note that the last value was converted to units of J from kJ in anticipation that joules would be the units required below.) We can use the equation below from the class notes to compute the characteristic length of the cylinder. Since we do not have any data for the length of the "long" cylinder we will assume that is the ratio D/L << 2 and can be neglected in computing the characteristic length.

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{4}D^2 L}{2\frac{\pi}{4}D^2 + \pi DL} = \frac{1}{\frac{2}{L} + \frac{4}{D}} = \frac{\frac{D}{2}}{2 + \frac{D}{L}} \approx \frac{D}{4} = \frac{2\,cm}{4} = 0.5\,cm = 0.005\,m$$

$$Bi = \frac{hL_c}{k} = \frac{200\,W}{m^2 \cdot {}^oC}(0.005\,m)\left(\frac{m \cdot {}^oC}{401\,W}\right) = 0.0025 < 0.1$$

Since the Biot number is less than 0.1, we can use the lumped parameter analysis. In such an analysis, the time to reach a certain temperature is given by the following equation.

$$t = -\frac{1}{b}\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) \qquad b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

From the data in the problem we can compute the parameter, b, and then compute the time for the rod to cool to 25^{o}C.

$$b = \frac{h}{\rho c_p L_c} = \frac{200\,W}{m^2 \cdot {}^oC}\frac{m^3}{8933\,kg}\frac{kg \cdot {}^oC}{385\,J}\frac{1}{0.0025\,m}\frac{1\,J}{W \cdot s} = \frac{0.01163}{s}$$

Here we have T_∞ = 20^{o}C, T = 25^{o}C, and T_i = 100^{o}C Substituting these values and the value for b just found gives the following result for the time.

$$t = -\frac{1}{b}\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -\frac{s}{0.01163}\ln\left(\frac{25^oC - 20^oC}{100^oC - 20^oC}\right) = \boxed{238\ s}$$

4-24 Stainless steel ball bearings (ρ = 8085 kg/m^3, k = 15.1 W/m·^{o}C, c_p = 0.480 kJ/kg·^{o}C, and α = 3.91x10^{-6} m^2/s) having a diameter of 1.2 cm are to be quenched in water. The balls leave the oven at a uniform temperature of 900^{o}C and are exposed to air at 30^{o}C for a while before they are dropped into the water. If the temperature of the balls is not to fall below 850^{o}C prior to quenching and the heat transfer coefficient in the air is 125 W/m^2·^{o}C, determine how long they can stand in the air before being dropped into the water.

First we compute the characteristic length and the Biot number to see if the lumped parameter analysis is applicable.

$$L_c = \frac{V}{A} = \frac{\frac{\pi}{6}D^3}{\pi D^2} = \frac{D}{6} = \frac{1.2\,cm}{6} = \frac{0.012\,m}{6} = 0.002\,m$$

$$Bi = \frac{hL_c}{k} = \frac{125\,W}{m^2 \cdot {}^oC}(0.002\,m)\left(\frac{m \cdot {}^oC}{15.1\,W}\right) = 0.0166 < 0.1$$

Since the Biot number is less than 0.1, we can use the lumped parameter analysis. In such an analysis, the time to reach a certain temperature is given by the following equation.

$$t = -\frac{1}{b}\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) \qquad b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

From the data in the problem we can compute the parameter, b, and then compute the time the ball bearings can remain in air before their temperature reaches 850°C.

$$b = \frac{h}{\rho c_p L_c} = \frac{125\,W}{m^2 \cdot {}^oC}\frac{m^3}{8085\,kg}\frac{kg \cdot {}^oC}{480\,J}\frac{1}{0.002\,m}\frac{1\,J}{W \cdot s} = \frac{0.01610}{s}$$

Here we have T_∞ = 30°C, T = 850°C, and T_i = 900°C Substituting these values and the value for b just found gives the following result for the time.

$$t = -\frac{1}{b}\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -\frac{s}{0.01610}\ln\left(\frac{850^oC - 30^oC}{900^oC - 30^oC}\right) = \boxed{\mathbf{3.68\ s}}$$

4-35 A student calculates that the total heat transfer from a spherical copper ball of diameter 18 cm initially at 200°C and its environment at a constant temperature of 25°C during the first 20 min of cooling is 3150 kJ. Is this result reasonable? Why?

As a reality check we can compute the maximum amount of heat transfer that would occur if the copper ball reached the ambient temperature. If the student's computed value is greater than this maximum value, the answer is wrong.

The maximum heat transfer can be found as the product of mass times heat capacity times the maximum temperature difference.

$$Q_{\max} = mc_p(T_i - T_\infty) = \rho V c_p(T_i - T_\infty) = \rho\frac{\pi D^3}{6}c_p(T_i - T_\infty)$$

We can find the density and heat capacity of copper from Table A-3 of the text: ρ = 8933 kg/m³ and c_p = 0.385 kJ/kg·°C = 385 J/kg·°C. With these properties we can find Q_{max} as follows.

$$Q_{\max} = \rho\frac{\pi D^3}{6}c_p(T - T_i) = \frac{8933\,kg}{m^3}\frac{\pi}{6}(0.18\,m)^3\frac{385\,J}{kg_m \cdot {}^oC}\left(200^oC - 25^oC\right) = 1838\,J$$

Since the student's answer is greater than the maximum it is ***not reasonable***!

4-37 An ordinary egg can be approximated as a 5.5-cmdiameter sphere whose properties are roughly k = 0.6 W/m²°C and α = 0.14x10⁻⁶ m²/s. The egg is initially at a uniform temperature of 8°C and is dropped into boiling water at 97°C. Taking the convection heat transfer coefficient to be h = 1400 W/m²·°C, determine how long it will take for the center of the egg to reach 70°C.

First we compute the characteristic length and the Biot number to see if the lumped parameter analysis is applicable.

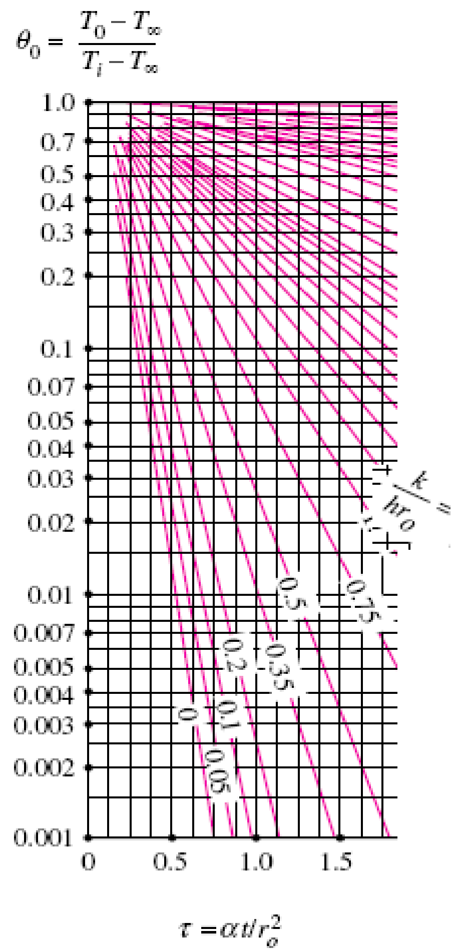
$$L_c = \frac{V}{A} = \frac{\frac{\pi}{6}D^3}{\pi D^2} = \frac{D}{6} = \frac{5.5\,cm}{6} = \frac{0.055\,m}{6} = 0.009167\,m$$

$$Bi = \frac{hL_c}{k} = \frac{1400\,W}{m^2 \cdot {}^oC}(0.009716\,m)\left(\frac{m \cdot {}^oC}{0.6\,W}\right) = 31.3 >> 0.1$$

Here, the lumped parameter analysis ***cannot be used*** so we have to use the charts. The chart for finding the temperature at the center of a sphere is Figure 4-17(a) on page 234 of the text. To use this chart we have to know two of the following three parameters: $(T_0 - T_\infty)/(T_i - T_\infty)$, k/hr_o, and $\alpha t/r_o^2$. In this problem, the unknown parameter is $\alpha t/r_o^2$; we can find this parameter from the chart and then use the known values of α and the outer radius, r_o, to find the time. The two known parameters are computed below.

$$\frac{k}{hr_o} = \frac{0.6\,W}{m\cdot{}^oC}\left(\frac{m^2\cdot{}^oC}{1400\,W}\right)\frac{1}{0.0275\,m} = 0.01558 \qquad \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{70^oC - 97^oC}{8^oC - 97^oC} = 0.303$$

An extract from the chart is shown at the right. There is no line for $k/hr_o = 0.01558$ but it would be between the first and second colored lines on the left of the chart (for $k/hr_0 = 0$ and 0.05). These two lines cross the horizontal line where the temperature ratio is 0.3 (close enough to our value of 0.303) where the horizontal axis has value of about 0.175; this is the value that the chart predicts for the dimensionless time τ, called the Fourier number: $\tau = \alpha t/r_o^2$. From this dimensionless time and the known values of α and r_o, we can find our desired answer: the time, t, required for the center of the egg (assumed spherical) to reach the temperature of 70°C.



$$t = \frac{\tau r_0^2}{\alpha} = \frac{0.175(0.0275\,m)^2}{\frac{0.14x10^{-6}\,m^2}{s}} = 945\,s$$

t = 15.8 min

This is a really hardboiled egg or the assumptions are not very good in this calculation.