California State University Northridge

College of Engineering and Computer Science
Mechanical Engineering Department

*Mechanical Engineering 375*
*Heat Transfer*

Spring 2007 Number 17629 Instructor: Larry Caretto

# February 28 Homework Solutions

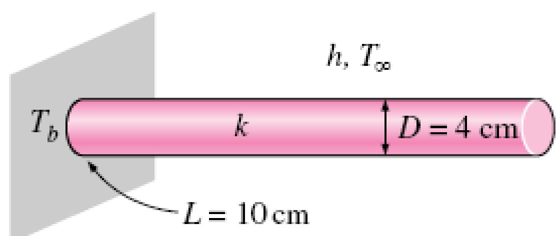
**3-111 The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 15 W is given to be 25°C/W. If the case temperature of the transistor is not to exceed 80°C, determine the power at which this transistor can be operated safely in an environment at 40°C.**

By the thermal circuit analogy, the heat transfer (which is the power dissipated by the transistor under its operation) is simply the temperature difference divided by the resistance.

$$\dot{Q} = \frac{\Delta T}{R} = \frac{80^o C - 40^o C}{\dfrac{25^o C}{W}} = 1.6\, W$$

The temperature conditions limit the operation of the transistor; an improved heat transfer operation is required for the transistor to operate at full power.

**3-112 A 4-mm-diameter and 10-cm-long aluminum fin (*k* = 237 W/m·°C) is attached to a surface. If the heat transfer coefficient is 12 W/m²·°C, determine the percent error in the rate of heat transfer from the fin when the infinitely long fin assumption is used instead of the adiabatic fin tip assumption**




The heat transfer formulas for the infinitely long fin and the fin with no heat transfer from the ends are shown below.

$$\left(\dot{Q}_{x=0}\right)_{\infty\ length} = \sqrt{kA_chp}\left(T_b - T_\infty\right) \qquad \left(\dot{Q}_{x=0}\right)_{no\ convection} = \sqrt{kA_chp}\left(T_b - T_\infty\right)\tanh mL$$

The relative error, $\varepsilon$, from using the formula for an infinitely long fin is then

$$\varepsilon = \left|\frac{\left(\dot{Q}_{x=0}\right)_{no\ convection} - \left(\dot{Q}_{x=0}\right)_{\infty\ length}}{\left(\dot{Q}_{x=0}\right)_{no\ convection}}\right| = \left|\frac{\sqrt{kA_chp}\left(T_b - T_\infty\right)\tanh mL - \sqrt{kA_chp}\left(T_b - T_\infty\right)}{\sqrt{kA_chp}\left(T_b - T_\infty\right)\tanh mL}\right| = \frac{1}{\tanh mL} - 1$$

In completing this formula we recognize that tanh(mL) is always less than 1 so the absolute value will be given by the difference shown. The problem statement and the accompanying figure have different dimensions for the fin diameter. We will first work the problem for the 4 mm diameter fin of the problem statement. For the pin fin, the cross sectional area is $A_c = \pi D^2/4 = \pi(0.004\text{ m})^2/4 = 1.257\text{x}10^{-5}\text{ m}^2$ and the perimeter is $p = \pi D = \pi(0.004\text{ m}) = 0.01257$ m. With these values and the given data for k and h we can compute the parameter m as follows.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$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{12\,W}{m^2\cdot{}^oC}\left(\frac{m\cdot{}^oC}{237\,W}\right)\frac{0.01257\,m}{1.257x10^{-5}\,m^2}} = \frac{7.11}{m}$$

Applying this value of m to our error formula with L = 10 cm = 0.1 m, so that mL = 0.711, gives.

$$\varepsilon = \frac{1}{\tanh 0.711} - 1 = 0.635$$

If the fin diameter were 4 cm = 0.04 m instead of 0.004 m, as shown in the diagram, the perimeter would increase by a factor of 10 and the area would increase a factor of 100 giving the following result.

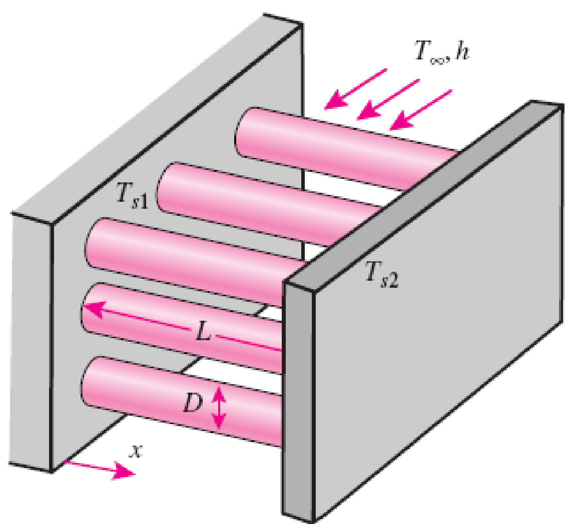
$$mL = L\sqrt{\frac{hp}{kA_c}} = (0.1\,m)\sqrt{\frac{12\,W}{m^2\cdot{}^oC}\left(\frac{m\cdot{}^oC}{237\,W}\right)\frac{0.1257\,m}{1.257x10^{-3}\,m^2}} = 0.2250$$

Applying this value of mL to our error formula gives.

$$\varepsilon = \frac{1}{\tanh 0.2250} - 1 = 3.52$$

These results show that the error for small values of mL can exceed 100%. Even an mL value around one can result in a large error.

**3-114 Circular cooling fins of diameter D = 1 mm and length L = 25.4 mm, made of copper (k = 400 W/m·K), are used to enhance heat transfer from a surface that is maintained at temperature $T_{s1}$ = 132°C. Each rod has one end attached to this surface (x = 0), while he opposite end (x = L) is joined to a second surface, which is maintained at $T_{s2}$ = 0°C. The air flowing between the surfaces and the rods is also at $T_\infty$ = 0°C, and the convection coefficient is h = 100 W/m²·K.**




**(a) Express the function u(x) = T(x) – $T_\infty$ along a fin, and calculate the temperature at x = L/2.**

**(b) Determine the rate of heat transferred from the hot surface through each fin and the fin effectiveness. Is the use of fins justified? Why?**

**(c) What is the total rate of heat transfer from a 10-cm by 10-cm section of the wall, which has 625 uniformly distributed fins? Assume the same convection coefficient for the fin and for the unfinned wall surface.**

The cooling fins in this example have a specified temperature at each end. No equation for this case is given in the text or the notes, however we can derive the necessary equation from the following equations for $\theta = T - T_\infty$: (a) the general result of equation (3-58) in the notes, $\theta(x) = C_1e^{mx} + C_2 e^{-mx}$, the boundary condition at the fin base, $\theta(0) - \theta_b$, and the boundary condition new to this problem that the value of $T - T_\infty$ is specified at x = L, $\theta(L) = \theta_L$. Substituting the two boundary conditions into the general solution and eliminating $C_1$ from the two equations gives.

$$\theta_b = C_1 e^0 + C_2 e^{-0} = C_1 + C_2 \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL} = (\theta_b - C_2)e^{mL} + C_2 e^{-mL}$$

Solving this for $C_2$ and then obtaining $C_1 = \theta_b - C_2$ gives

$$C_2 = \frac{\theta_L - \theta_b e^{mL}}{e^{-mL} - e^{mL}} \qquad C_1 = \theta_b - C_2 = \theta_b - \frac{\theta_L - \theta_b e^{mL}}{e^{-mL} - e^{mL}} = \frac{\theta_b\left(e^{-mL} - e^{mL}\right) - \theta_L + \theta_b e^{mL}}{e^{-mL} - e^{mL}} = \frac{\theta_b e^{-mL} + \theta_L}{e^{-mL} - e^{mL}}$$

We can substitute these equations for $C_1$ and $C_2$ into our general solution, $\theta(x) = C_1e^{mx} + C_2 e^{-mx}$, giving

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} = \frac{\theta_b e^{-mL} + \theta_L}{e^{-mL} - e^{mL}} e^{mx} + \frac{\theta_L - \theta_b e^{mL}}{e^{-mL} - e^{mL}} e^{-mx}$$

For a pin fin, the ratio $A_c/p = (\pi D^2/4) / (\pi D) = D/4$, which is (0.001 m)/4 = 0.00025 m in this problem. With this result and the other the data in this problem, we can find the parameters, m and mL, as follows:

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{100\,W}{m^2 \cdot {}^oC}\left(\frac{m \cdot {}^oC}{400\,W}\right)\frac{1}{0.00025\,m}} = \frac{31.62}{m} \qquad mL = \frac{31.62}{m}(0.0254\,m) = 0.8032$$ For

From the temperatures given in this problem we have the problem given here, $\theta_b = 132^oC - 0^oC = 132^oC$ and $\theta_L = 0^oC - 0^oC = 0^oC$. We can substitute these values of mL, $\theta_b$ and $\theta_L$ to find the temperature at x = L/2 = 0.-127 m (mL = 0.4016)

$$\theta(L/2) = \frac{\theta_b e^{-mL} + \theta_L}{e^{-mL} - e^{mL}} e^{mL/2} + \frac{\theta_L - \theta_b e^{mL}}{e^{-mL} - e^{mL}} e^{-mL/2} = \frac{(132^o\,C)e^{-0.8032} + 0^o\,C}{e^{-0.8032} - e^{0.8032}} e^{0.4016} + \frac{0^o\,C - (132^o\,C)e^{-0.8032}}{e^{-0.8032} - e^{0.8032}} e^{-0.4016}$$

Since $T_\infty = 0$, $\boxed{T(L/2) = \theta(L/2) = 61.0^oC}$

(b) The heat transfer is found by taking the derivative of the equation for $\theta$.

$$\dot{q} = -k\frac{dT}{dx} = -k\frac{d\theta}{dx} = -k\left[\frac{\theta_b e^{-mL} + \theta_L}{e^{-mL} - e^{mL}} m e^{mx} + \frac{\theta_L - \theta_b e^{mL}}{e^{-mL} - e^{mL}}\left(-m e^{-mx}\right)\right]$$

At x = 0, we find the heat flux as follows.

$$\dot{q}_{x=0} = -km\left[\frac{\theta_b e^{-mL} + \theta_L}{e^{-mL} - e^{mL}} - \frac{\theta_L - \theta_b e^{mL}}{e^{-mL} - e^{mL}} e^{-mx}\right] =$$

$$-\frac{400\,W}{m \cdot {}^oC}\frac{31.62}{m}\left[\frac{(132^o\,C)e^{-0.8032} + 0^o\,C}{e^{-0.8032} - e^{0.8032}} - \frac{0^o\,C - (132^o\,C)e^{-0.8032}}{e^{-0.8032} - e^{0.8032}}\right] = \frac{2.508x10^6\;W}{m^2}$$

If there were no fin, all the heat transfer would be by convection

$$\dot{q}_{conv} = h(T_b - T_\infty) = \frac{100\,W}{m^2 \cdot {}^oC}(132^oC - 0^oC) = \frac{13200\,W}{m^2}$$

The fin effectiveness is the ratio of the actual fin heat transfer to the heat transfer that would occur with no fin.

$$\varepsilon_{fin} = \frac{\dot{q}_{x=0,fin}}{\dot{q}_{conv}} = \frac{\frac{2.508x10^6\ W}{m^2}}{\frac{13200\ W}{m^2}} \qquad \boxed{\varepsilon_{fin} = 192}$$

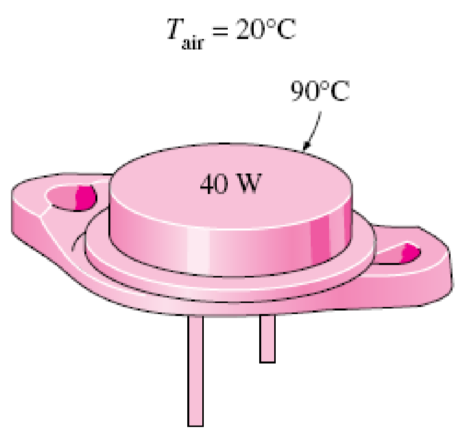
A fin is usually justified if the effectiveness is greater than 2. It is certainly justified in this case.

(c) The total heat transfer in this case is the sum of two components: (1) the heat flux from one fin, times the area of one fin, $A_{fin} = \pi(0.001\ m)^2/4 = 7.854x10^{-7}$, times the number of fins, $n_{fin}$ = 625 fins, and (2) the heat flux from the unfinned area, $A_{unfin}$. = (0.1 m)(0.1 m) – 625$A_{fin}$. Both of there heat fluxes were found in part (b), so we can obtain the total heat transfer as follows.

$$\dot{Q} = \dot{q}_{conv}A_{unfin} + \dot{q}_{x=0,fin}n_{fin}A_{fin} = \left[.01\ m^2 - 625\left(7.854x10^{-7}\ m^2\right)\right]\frac{13200\ W}{m^2} + 625\left(7.854x10^{-7}\ m^2\right)\frac{2.508x10^6\ W}{m^2}$$

$$\boxed{\dot{Q} = 1363\ W}$$

**3-115 A 40-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–6. Select a heat sink that will allow the case temperature of the transistor not to exceed 90°C in the ambient air at 20°C.**

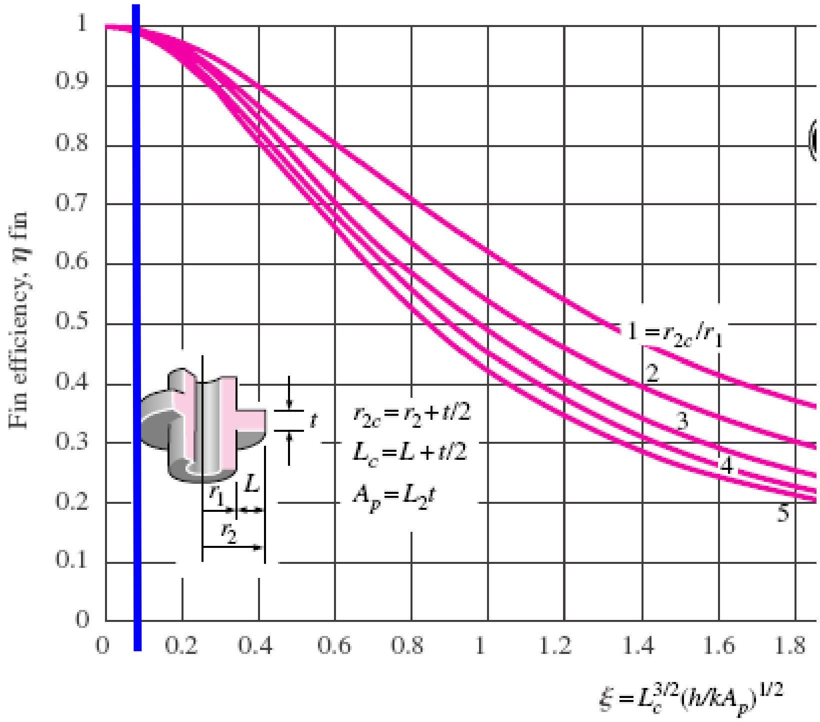



This problem is similar to problem 3-111 where we were given a temperature difference and a resistance and had to find the heat transfer. Here we have the heat transfer and the temperature difference and we have to find the resistance. Using the thermal circuit Ohm's law analogy as an inequality – we want to have a resistance that just matches the Ohm's law value or a smaller one – gives the following result.

$$R \le \frac{\Delta T}{\dot{Q}} \le \frac{90^o C - 20^o C}{40\ W} = \frac{1.75^o C}{W}$$

Looking at Table 3-6 we see that the following heat sinks have the required resistance: HS 5030 (either horizontal or vertical) HS 6071 (vertical) and HS 6115 (either horizontal or vertical). Final selection would be based on other factors on cost, space requirements, and integration with other design components.

**3-117 Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. Circular aluminum alloy 2024-T6 fins (k = 186 W/m·°C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty$ = 25°C, with a heat transfer coefficient of 40 W/m²·°C. Determine the increase in heat transfer.**




We do not have an equation for the fin ***effectiveness*** in this case, but we do have a chart and an equation for the fin ***efficiency***. The efficiency equation for a single fin is given in Table 3-3 on page 165; the efficiency is shown in Figure 3-43 on page 167. Part of this chart is copied at the left. Both the chart and the equation use the nomenclature for the fin dimensions shown in the copy of the chart at the right. (There is a typographical error in the chart $L_2$ should be $L_c$.)

For the fin in this problem, $r_1$ = 0.025 m, $r_2$ = .03 m, t = 0.001 m, and L = $r_2 - r_1$ = 0.005 m. The other parameters shown in the figure can be computed as follows: $r_{2c} = r_2 + t/2$ = 0.03 m + (0.001 m)/2 = 0.0305 m; $L_c$ = L + t/2 = 0.005 m + (0.001 m)/2 = 0.0055 m; $A_p = L_c t$ = (0.0055 m)(0.001 m) = 5.5x10$^{-6}$ m$^2$. We can now compute the parameters required to find the efficiency in Figure 3-43.

$$\frac{r_{2c}}{r_1} = \frac{0.0305\,m}{0.025\,m} = 1.22 \qquad \xi = \sqrt{\frac{L_c^3 h}{kA_p}} = \sqrt{\frac{(0.0055\,m)^3 \dfrac{40\,W}{m^2\cdot{}^oC}}{\dfrac{186\,W}{m\cdot{}^oC}\left(5.5x10^{-6}\,m^2\right)}} = 0.081$$

This value of $\xi$ is plotted as the blue vertical line on the chart. All the lines for $r_{2c}/r_1$ merge at this value of $\xi$ and the value of the fin effectiveness is about 0.98. The fin heat transfer is this value times the maximum heat transfer that would occur if the entire fin area was at the base temperature of 180$^o$C. The fin area is the area of the two surfaces plus the end of the fin.

$$A_{fin} = 2\pi\left[(0.03\,m)^2 - (0.025\,m)^2\right] + \pi(0.06\,m)(0.001\,m) = 0.001916\,m^2$$

$$\dot{Q}_{\max} = hA_{fin}(T_b - T_\infty) = \frac{40\,W}{m^2\cdot{}^oC}\left(0.001916\,m^2\right)\left(180^oC - 25^oC\right) = 11.88\,W$$

The actual heat transfer is 0.98(11.88 W) = 11.64 W. In 1 m of tube length there are 250 fins with a thickness of 0.001 m and a spacing between fins of 0.003 m Thus there is an area of 250$\pi$(.05 m)(.003 m) = 0,1178 m$^2$. The heat transfer from this area is

$$\dot{Q}_{no\ fin} = hA_{no\ fin}(T_b - T_\infty) = \frac{40\,W}{m^2\cdot{}^oC}\left(0.1178\,m^2\right)\left(180^oC - 25^oC\right) = 730.4\,W$$

Adding this heat transfer to the 11.64 W transferred from each fin gives the actual heat transfer as

$$\dot{Q} = \dot{Q}_{no\ fin} + \dot{Q}_{fin} = 730.4\,W + 250(11.64\,W)\ \boxed{\boldsymbol{= 3641\ W}}$$

The increase per meter of length is 3641 W – (4/3)(730.4 W) = 2667 W, an increase of 274%

The efficiency equation, which is copied from Table 3-3 in the text below, uses modified Bessel functions. These can be found in Table 3-4 on page 166 of the text or by using functions available in Excel and Matlab.

**Circular fins of rectangular profile**

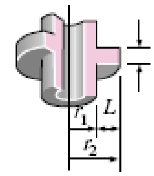
$m = \sqrt{2h/kt}$

$r_{2c} = r_2 + t/2$

$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$

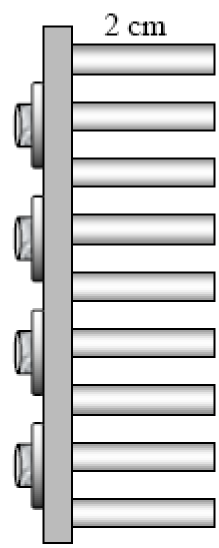
$$\eta_{\text{fin}} = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$




Using the equation gives a fin effectiveness of 0.9952 which would predict a slightly greater heat transfer than the value of 0.98 read from the chart.

**3-121 A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.04 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 30 W/m·°C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 40°C, with a heat transfer coefficient of 40 W/m²·°C.**




**(a) Determine the temperatures on the two sides of the circuit board.**

**(b) Now a 0.2-cm-thick, 12-cm-high, and 18-cm-long aluminum plate (k = 237 W/m·°C) with 864 2-cm-long aluminum pin fins of diameter 0.25 cm is attached to the back side of the circuit board with a 0.02-cm-thick epoxy adhesive (k = 1.8 W/m·°C). Determine the new temperatures on the two sides of the circuit board.**

(a) With only the circuit board, we have a system with two resistances (circuit board conduction, R = L/kA, and convection, R = 1/hA). The area, A = (0.12 m)(0.18 m) = 0.0216 m². The heat transfer is the 80(0.04 W) = 3.2 W dissipated by the logic chips. The heat transfer between the unknown circuit-board-front-side temperature, $T_0$, and the ambient temperature of 40°C can be solved for the unknown temperature.

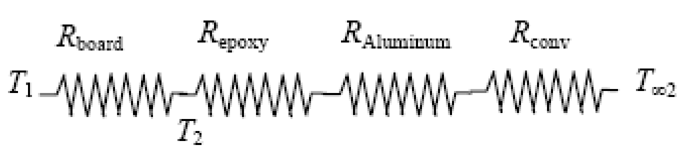
$$\dot{Q} = \frac{T_1 - T_\infty}{\frac{L}{kA} + \frac{1}{hA}} \quad \Rightarrow \quad T_1 = T_\infty + \frac{\dot{Q}}{A}\left(\frac{L}{k} + \frac{1}{h}\right) = 40^o C + \frac{3.2\,W}{0.0216\,m^2}\left(\frac{0.003\,m}{\frac{30\,W}{m\cdot{}^oC}} + \frac{1}{\frac{40\,W}{m^2\cdot{}^oC}}\right) = 43.72^o C$$

We can now analyze the conduction across the cirtuit board to find the temperature on the other side.

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L} \quad \Rightarrow \quad T_2 = T_1 - \frac{L\dot{Q}}{kA} = 43.72^o C + \frac{(0.003\,m)(2.4\,W)}{\frac{30\,W}{m\cdot{}^oC}(0.0216\,m^2)} = 43.70^o C$$

So the temperatures on the two sides of the circuit board are almost the same.

(b) With the epoxy layer, aluminum plate and fins added, we now have additional resistances. We can consider the fins as a way to reduce the convection



resistance, so we will analyze the fin heat transfer as part of the convective resistance. This gives the equivalent circuit shown above.

The convection resistance in this case includes the heat transfer from the fins plus the heat transfer from the unfinned area. To treat this as a resistance, we can write the equation for the heat transfer from the finned surface in an equivalent circuit form to deduce the resistance. This total heat transfer is given by the following equation.

$$\dot{Q} = \dot{Q}_{fin} + \dot{Q}_{no\ fin} = \eta_{fin} hA_{fin}(T_b - T_\infty) + hA_{no\ fin}(T_b - T_\infty)$$

Writing this in terms of an Ohm's law expression gives.

$$\dot{Q} = \frac{T_b - T_\infty}{\eta_{fin} hA_{fin} + hA_{no\ fin}} = \frac{T_b - T_\infty}{R_{fin}} \quad \Rightarrow \quad R_{fin} = \frac{1}{\eta_{fin} hA_{fin} + hA_{no\ fin}}$$

We can compute the first three individual resistances for our thermal circuit.

$$R_{board} = \frac{L_{board}}{k_{board} A} = \frac{0.003\ m}{\frac{30\ W}{m \cdot {}^oC}\left(0.0216\ m^2\right)} = \frac{0.00463\ {}^oC}{W}$$

$$R_{epoxy} = \frac{L_{epoxy}}{k_{epoxy} A} = \frac{0.0002\ m}{\frac{.18\ W}{m \cdot {}^oC}\left(0.0216\ m^2\right)} = \frac{0.0051\ {}^oC}{W}$$

$$R_{Al} = \frac{L_{Al}}{k_{Al} A} = \frac{0.002\ m}{\frac{237\ W}{m \cdot {}^oC}\left(0.0216\ m^2\right)} = \frac{0.00039\ {}^oC}{W}$$

To compute the resistance for the fins, we first have to compute the m parameter which depends on the fin dimensions: a diameter of 0.0025 m and a length of 0.02 m. The cross sectional area and perimeter of the fins are $A_c = \pi(0.0025\ m)^2/4 = 4.909 \times 10^{-6}\ m^2$ and $p = \pi(0.0025\ m) = 0.007854\ m$. The corrected length, $L_c = L + A_c/p = 0.02\ m + (4.909 \times 10^{-6}\ m^2)/(0.007854\ m) = 0.020625\ m$. The total fin area is $pL_c = (0.007854\ m)(0.020625\ m) = 1.620 \times 10^{-4}\ m^2$. With these dimensions and the values given for h and k we can find the m parameter as follows.

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{40\ W}{m^2 \cdot {}^oC}\left(\frac{m \cdot {}^oC}{237\ W}\right)\frac{0.007854\ m}{4.909 x 10^{-6}\ m}} = \frac{16.43}{m} \qquad mL_c = \frac{16.43}{m}\left(0.020625\ m\right) = 0.3389$$

The efficiency of a constant cross section fin is given by the following equation.

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c} = \frac{\tanh(0.3389)}{0.3389} = 0.9634$$

The total area of the aluminium block is 0.0216 $m^2$; the area occupied by fins cross section is $(864)(4.909 \times 10^{-6}\ m^2) = 0.004398\ m^2$. The unfinned area then is $0.0216\ m^2 - 0.004398\ m^2 = 0.01720\ m^2$. The total exposed area of the fins is $(896)(1.620 \times 10^{-4}\ m^2) = 0.1451\ m^2$. We now have the data required to compute the fin resistance.

$$R_{fin} = \frac{1}{h\left(\eta_{fin} A_{fin} + A_{no\ fin}\right)} = \frac{1}{\frac{40\ W}{m^2 \cdot {}^oC}\left[0.9634\left(0.1451\ m^2\right) + 0.01720\ m^2\right]} = \frac{0.1592\ {}^oC}{W}$$

The heat transfer is now found as the overall temperature difference divided by the total resistance.

$$\dot{Q} = \frac{T_1 - T_\infty}{\frac{L_{board}}{k_{board}A} + \frac{L_{epoxy}}{k_{epoxy}A} + \frac{L_{Al}}{k_{Al}A} + R_{fin}} \quad \Rightarrow \quad T_1 = T_\infty + \dot{Q}\left(\frac{L_{board}}{k_{board}A} + \frac{L_{epoxy}}{k_{epoxy}A} + \frac{L_{Al}}{k_{Al}A} + R_{fin}\right)$$

$$= 40^oC + 3.2\,W\left(\frac{0.00463^oC}{W} + \frac{0.0051^oC}{W} + \frac{0.00039^oC}{W} + \frac{0.1592^oC}{W}\right) = 40.54^oC$$

As we did in part (a), we can now analyze the conduction across the circuit board to find the temperature on the other side.

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L} \quad \Rightarrow \quad T_2 = T_1 - \frac{L\dot{Q}}{kA} = 40.54^oC + \frac{(0.003\,m)(2.4\,W)}{\frac{30\,W}{m \cdot {}^oC}(0.0216\,m^2)} = 40.53^oC$$

The temperature difference across the circuit board has not changed. The heat flux and the resistance across the board are the same as in part (a). Although the temperatures are lower now, the computed difference $T_1 - T_2 = 0.0148^oC$ remains the same in both parts.